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SEDIMENTATION STUDIES IN THE WESTERN GULF STATES[a]

Graham W. Renfro,[1] and Charlie M. Moore,[2] A.M. ASCE
(Proc. Paper 1806)

SUMMARY

A summary of some of the more important reservoir sedimentation surveys made by the Soil Conservation Service in the area is presented. An analysis is made of existing suspended sediment and reservoir sedimentation survey records in a seven-state area which includes 26 physiographic areas. Detailed methods used by the Soil Conservation Service in determining sediment yields are described. The effect of watershed protection measures, including floodwater retarding structures in reducing sediment yield from watersheds is discussed.

INTRODUCTION

Methods of making reservoir sedimentation surveys have been discussed in detail in previously published proceedings(7) and will therefore not be discussed further. In surveys now being made by the Soil Conservation Service emphasis is placed on determination of the factors influencing sediment yield. It is important to understand the effect of each factor on sediment yield in order to obtain improved bases for predicting yields from unmeasured watersheds. This paper summarizes some results of both reservoir and suspended sediment measurements to illustrate how such data are used for purposes of planning, design and construction.

Note: Discussion open until March 1, 1959. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1806 is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. HY 5, October, 1958.

a. Presented at the February 1957 ASCE Convention in Jackson, Miss.

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Summary of Reservoir Surveys

Detailed and reconnaissance sedimentation surveys have been made on approximately 150 reservoirs in the Western Gulf area, comprising the states of Arkansas, Louisiana, Oklahoma and Texas. Results of 16 surveys, representing 16 distinct physiographic areas, are shown in Table 1. These surveys were made by either the range or contour methods described by Eakin.(3)

The following factors have a marked influence on rates of capacity loss in storage reservoirs:

1. Capacity-watershed ratio. This is the ratio of the original storage of the reservoir per square mile of drainage area. In general, if this ratio is 100 acre-feet per square mile or more a reasonably long life for the reservoir is assured. As an example, Wilson Reservoir has a C/W ratio of 227 acre-feet/square mile. Its annual rate of capacity loss is only 0.11 percent.

2. The amount of sediment inflow to the reservoir or sediment yield. Several factors determine this yield, which is the function of the amount of gross erosion in the watershed and the efficiency of the stream system in transporting the erosional material and stream bed load material to the reservoir site.(4) Other factors which directly affect this yield are: size, shape and topography of the watershed, channel density, stream gradients, amount, intensity, and frequency of both rainfall and runoff, inherent erodibility of the soil, length and degree of land slopes, land use, kind and amounts of vegetative cover and effect of conservation practices applied in the watershed.

3. Trap efficiency of the reservoir. This is expressed as the ratio between sediment accumulation and sediment outflow and is dependent on such factors as the ratio between storage capacity and inflow, age of the reservoir, shape of the reservoir basin, type and method of operation of the outlets, which affects detention storage time, texture of incoming sediment and behavior of fine sediment fractions under various conditions.

In general a reservoir with a high capacity-watershed ratio will have a high trap efficiency since the volume of water passing through the reservoir will be small. Conversely, a low trap efficiency would normally occur in reservoirs with low C/W ratios. Exceptions to both rules can be found. For example a high trap efficiency might occur in reservoirs of an arid region where spillways seldom, if ever, discharge. In contrast, a reservoir with the same C/W ratio in a very humid area might have a comparatively low trap efficiency due to frequent, sustained flows through its spillway. Recent studies indicate that use of the capacity-inflow ratio (C/I) offers better criteria to estimate trap efficiency than the capacity-watershed ratio (C/W).

Analysis of Sediment Records

During the years 1954-1955, a detailed study was undertaken by the Soil Conservation Service, Engineering and Watershed Planning Unit, Fort Worth, Texas in an attempt to bring together in one publication all known existing records on rates of sediment production in the Western Gulf states.(3)

These records include 75 measurements of suspended sediment and 179 reservoir sedimentation surveys. Table 2 shows the distribution of these sediment records among the 26 physiographic areas.

The following agencies, in addition to SCS, furnished data on suspended sediment measurements and reservoir sedimentation surveys.

Table 1

SEDIMENTATION DATA ON RESERVOIRS IN THE WESTERN GULF STATES

Name of Reservoir	Location	Date Storage Began	Watershed Area	Reservoir Area	Original Capacity of Reservoir	Original Storage per sq.mi. dr. area	Annual Sed. Production per sq.mi. dr. area	Annual Depletion of Storage	Physiographic Province	Type of Survey
			(sq.mi.)	(acres)	(ac.ft.)	(ac.ft.)	(ac.ft.)	(percent)		
Lake Bailey	Morrilton, Ark.	1937	15	115	629	42	.89	2.12	Ark. Valley	Detailed
Lake Winona	Little Rock, Ark.	1937	43	1,085	34,533	803	.37	.04	Ouachita Mts.	Reconn.
Wilson Reservoir	Fayetteville, Ark.	1930	2.3	29	522	227	.24	.11	Ozark Mts.	Detailed
Loring Lake	Zwolle, La.	1928	1.0	64	663	663	1.70	.26	Forested Coastal Plain	Reconn.
Lake Clinton	Clinton, Okla.	1930	23.1	336	4,415	191	2.70	1.41	Red Hills	Detailed
Lake Guthrie	Guthrie, Okla.	1920	12.9	226	3,064	237	3.02	1.27	Red Bed Plains	Detailed
Brown Lake	McAlester, Okla.	1943	19.9	637	4,924	247	1.91	.77	Cherokee Pr.	Detailed
Wetumka Lake	Wetumka, Okla.	1931	3.9	175	2,076	532	2.02	.38	Sandstone Hills	Reconn.
Baker Lake	Somerset, Texas	1950	3.1	46	251	81	1.55	1.91	So. Texas Coastal Plain	Reconn.
Cottonwood Res.	Fabens, Texas	1938	2.3	88	417	181	1.23	.67	Basin & Range Province	Detailed
Lake Fryer	Perryton, Tex.	1939	108	102	844	8	.17	2.17	High Plains	Reconn.
Lake Dallas	Dallas, Texas	1928	1167	10,903	180,759	155	1.18	.76	Grand Prairie	Detailed
Eagle Mt. Lake	Fort Worth, Tex.	1934	809	9,847	420,000	519	1.50	.29	West Cross Timbers	Reconn.
Medina Lake	San Antonio, Tex.	1913	578	5,647	274,065	474	.45	.09	Edwards Plateau	Detailed
Variety Cl. Lake	Bedford, Texas	1942	.3	8	33	110	2.40	2.18	East Cross Timbers	Reconn.
White Rock Lake	Dallas, Texas	1910	97.1	1,254	18,158	187	1.67	.89	Black Prairie	Detailed

Table 2
Distribution of Sediment Records
Western Gulf States

Physiographic Area	Suspended Sediment Measurements	Reservoir Sedimentation Surveys	Total
Arbuckle Mountains	1	0	1
Arkansas Valley	2	5	7
Basin and Range Province	0	4	4
Blackland Prairies	15	25	40
Cherokee Prairies	2	7	9
Coastal Prairie	3	0	3
East Cross Timbers	0	1	1
Edwards Plateau	1	5	6
Flint Hills	5	4	9
Forested Coastal Plain	2	38	40
Grand Prairie	1	2	3
High Plains	1	1	2
Lampasas Cut Plain	1	5	6
Llano Basin	0	4	4
Mississippi Alluvial Valley	1	0	1
Ouachita Mountains	9	2	11
Ozark Mountains	8	7	15
Palo Pinto Section	0	7	7
Pecos Valley	0	1	1
Pine Flats	1	0	1
Red Bed Plains	13	15	28
Red Hills	6	29	35
Sandstone Hills	2	6	8
South Texas Coastal Plain	1	3	4
Southern Rocky Mountains	0	2	2
West Cross Timbers	0	6	6
Total	75	179	254

1. Agricultural Research Service, Blacklands Experimental Watershed, Riesel, Tex.
2. U. S. Geological Survey, at Fayetteville, Ark.; Oklahoma City, Okla.; and Austin, Tex.
3. U. S. Corps of Engineers, at Tulsa, Okla.; Vicksburg, Miss.; Little Rock, Ark.; Galveston, Tex.; and Fort Worth, Tex.
4. U. S. Bureau of Reclamation, Amarillo, Tex.
5. State of Texas, Board of Water Engineers, Austin, Tex.

Suspended-sediment records were in most cases summarized by the flow-duration curve and sediment-rating curve method. Usually at least 25 sediment samples covering a wide range of water discharges in different seasons of the year were required. Normally, 10 years of water records were used to compile the flow-duration curve for each station. Suspended sediment records from the Blacklands Experimental Watershed, however, were computed by the sediment-hydrograph method, as very detailed sampling was available for these records.

Adjustments were made in the records for (1) type of sampler used and (2) unmeasured bed load. Using all of the records selected, the adjusted annual rate of sediment production in tons was plotted against sediment-contributing area in square miles. There was considerable spread in the points on the graph, because of the many factors which were not evaluated, such as land use, slope, conservation practices, relief, channel density, and runoff. However, there was a distinct trend toward lower rates of sediment production with increasing sizes of drainage area.

Further details concerning analysis and use of these records are contained in reference number 2, "Rates of Sediment Production in the Western Gulf States" by Brune, Maner, Renfro and Ogle.

Because of the scarcity of records in some of the physiographic areas (Fig. 1) additional records from the states of Mississippi, Kansas and New Mexico were included in the study. The Mississippi records were particularly valuable since they are applicable to the Forested Coastal Plain physiographic area in Louisiana where few sediment records exist. Because of length of the table giving detailed information on these 254 records it is not included here.

Adjustments were made for estimated trap efficiency of reservoirs. For suspended sediment records adjustments were made for (1) type of sampler used and (2) estimates for bedload, which represented an average of 10-15 percent of the total sediment load.

In addition to its value in having all sediment records summarized in one publication, the study provides a means for making general estimates of rates of sediment production by physiographic areas. Such estimates are necessary in preliminary design and construction phases of watershed planning by the Soil Conservation Service, which includes various types of reservoirs. In addition it provides approximate information to State and Federal agencies and consulting engineers for planning municipal, power or recreational reservoirs and to local agencies on the sediment load of a particular stream or the capacity loss of a specific reservoir.

Additional studies covering individual physiographic areas have been made and more are contemplated. These studies are in more detail and are for the purpose of determining the relationship between sediment production and watershed causal factors such as length and percent of slope, land use, channel density, relief and conservation treatment.

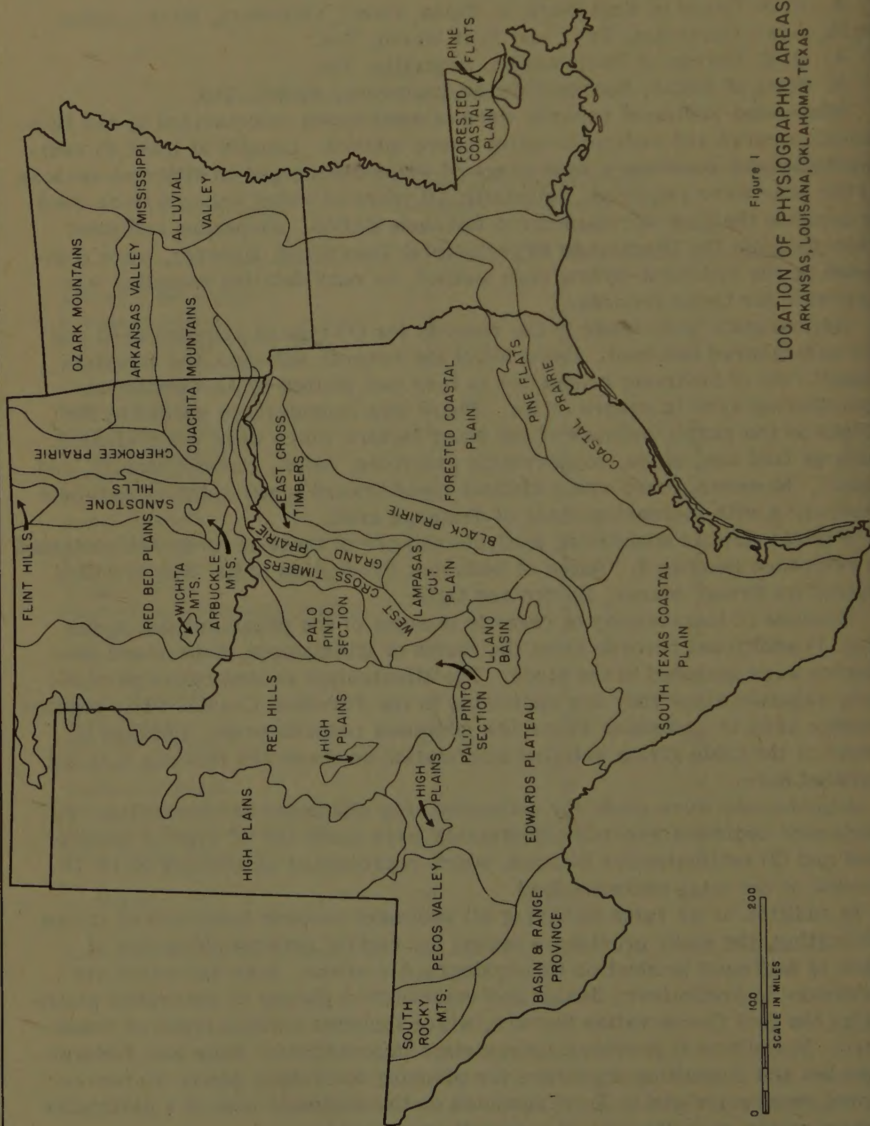

Figure 1

LOCATION OF PHYSIOGRAPHIC AREAS
ARKANSAS, LOUISANA, OKLAHOMA, TEXAS

The two detailed studies completed thus far are in the Black Prairie and Red Hills[8] physiographic areas. These studies permit a more accurate prediction of rates of sediment production and distribution of sediment in proposed reservoirs.

Methods Used by the Soil Conservation Service in Western Gulf States To Determine Sediment Yields for Design Purposes

In designing a floodwater retarding structure, it is important that an accurate estimate be made of the sediment storage requirements. To assure adequate useful life of floodwater retarding structures, sediment pools are designed to have a minimum life of 50 years. Under-estimation of the average annual sediment yield will result in a depletion of the sediment pool in less than 50 years, and consequent encroachment on the detention pool. Over-estimation results in excessive costs.

As pointed out previously, the factors which govern the processes of sedimentation in reservoirs are known. All these factors can be measured or estimated closely.

Detailed sediment yield studies are designed to attain two immediate objectives (1) to determine as accurately as possible, by using erosional measurements and estimates, the total annual sediment yield to the proposed structure and (2) to determine the distribution and volume of the sediment yield retained within the pools of the structures.

In estimating annual gross erosion the quantity of material derived from sheet erosion and the quantity derived from channel erosion are computed separately.

Sheet Erosion

Sheet erosion is computed by use of the Musgrave[9] equation which is expressed as:

$E = FRS^{1.35} L^{0.35} P_{30}{}^{1.75}$, where

E = Sheet erosion, inches per year

F = Soil factor, basic erosion rate, inches per year, for each soil unit

R = Cover factor

S = Degree of land slope, in percent

L = Length of land slope, in feet

P_{30} = Maximum 30-minute rainfall, in inches, 2-year frequency.

Basic erosion rates have been established on all soil units occurring in the Western Gulf states. These soils were classified into groups with similar family characteristics and similar erosion potentialities. This classification constitutes a useful basis for evaluating erodibility of soils since erosion rates of many soils, representing large groups of families, have been evaluated at experiment stations within the area.

The erosion rates are based on 10 percent slope, 72.6 feet slope length, 100 percent row crop cover and a maximum 30-minute rainfall intensity of 1.375 inches occurring once in two years.

The above set of conditions is adjusted for actual field conditions of slope

percent, slope length, rainfall intensity, crop rotations and cover which occur in the particular watershed under study.

Tables have been developed showing: (1) basic erosion rates, in inches of annual soil loss, for all soil units; (2) adjustment factors for slope percent and length; (3) adjustments for various types of cropland cover; (4) adjustments for all degrees of range and woodland cover; and (5) adjustments for rainfall occurrence frequency within the various rainfall belts.

In order to compute the volume of gross sheet erosion, using the method developed by Musgrave and associates, it is necessary to determine from a watershed survey the following:

1. Total acres in each soil unit by percent slope, slope length in feet and land use.
2. Rotations used on cultivated land (percent of each crop grown).
3. Cover condition classes on all pasture, meadow or woodland.
4. Land use capability of the above separations.
5. Maximum 30-minute rainfall intensity occurring once in two years.

Soil surveys have been made on a high percentage of the total land area in each of the Western Gulf states. From these, soil units, slope percent and land use are tabulated. Additional field work is required to obtain slope length data, crop rotations and cover conditions.

Rainfall data is based on USDA Miscellaneous Publication No. 204, "Rainfall Intensity - Frequency Data" by David L. Yarnell.

The computation of volume of material derived from sheet erosion is made by use of the tables referred to on page 10.

A sample calculation in estimating the annual rate of gross sheet erosion for a given set of conditions is as follows:

Given:

Cultivated land	767 acres
Soil unit	1
Slope	3 percent
Slope length	350 feet
Average land use or cover	90 percent row crop; 10 percent fall-planted small grain
Rainfall	1.375 inches (30-minute rainfall intensity, 2-year frequency)

Problem: Compute the average annual rate of soil loss, in acre-feet, from sheet erosion.

Solution:
1. Given, 767 acres of cultivated land.
2. The basic erosion rate for soil unit 1 is 0.65", annual soil loss.
3. The adjustment factor for a three percent slope on a 350-foot slope length, is 0.335.
4. The land use adjustment factor for 90 percent row crop and 10 percent fall-planted small grain is 0.93.
5. The rainfall adjustment factor for a 1.375" P_{30} 2-year frequency rain is 1.0.

Computing the product of the above values: 767 (acres cultivated) x 0.65"

(basic erosion rate of soil unit 1) x 0.335 (adjustment factor for 3 percent slope and 350-foot slope length) x 0.93 (adjustment factor for 90 percent row crop and 10 percent fall-planted small grain) x 1.0 (adjustment factor for a P_{30} rainfall intensity of 1.375", 2-year frequency) = 155.32 acre-inches. 153.00" ÷ 12 = 12.94 acre-feet, annual soil loss.

The same procedure is used to compute annual soil loss rates for pasture and woodland.

Channel Erosion

Channel erosion includes gully erosion, valley trenching and streambed and streambank erosion. Measurements of soil loss by these processes are usually made and recorded in the field. Annual rates of lateral or vertical cutting for various types of channel erosion are estimated by comparing aerial photographs of different dates, from comparison of cross section surveys, or from historical data obtained from landowners or operators.

The formula for computing channel erosion is given below:

$$D \times L \times R \times N \times P \div 43{,}560 = \text{acre-feet of sediment where,}$$

D = Depth of channel, in feet

L = Length of channel, in feet

R = Annual rate of lateral erosion, in feet

N = Number of banks affected

P = Percent of bank length eroding

Road erosion is a significant source of sediment in many areas within the Western Gulf states. Sediment production from this source can be computed by two methods:

1. In the same manner as gully erosion.
2. By use of the formula:

$$\frac{D}{Y} \times W \times L \div 43{,}560 = \text{acre-feet of sediment, where}$$

D = Depth of roadbed removal, in feet

Y = Age of road in years

W = Width of road surface, in feet

L = Length of eroded section of road, in feet.

Rates of Sediment Delivery

Sediment delivery rate, defined as the ratio between annual sediment yield and annual gross erosion[4] is dependent upon a number of watershed factors. Among these are size and shape of the net sediment-contributing area, channel density, and the relationship between certain watershed characteristics such as topography, which may be expressed quantitatively by the relief-length (R/L) ratio.

Sediment delivery rates have been investigated in detail in two important physiographic areas in the Western Gulf states. MANER and BARNES[1] in a study of the Texas Blackland Prairies found watershed area to be an excellent indicator of the sediment delivery rate within a given size range. In

this study, results of detailed reservoir sedimentation surveys were correlated with detailed studies of watershed sediment sources. Based on these data the average annual volume, in acre-feet, of sediment derived from sheet erosion reaching each reservoir was obtained by making adjustments for sediment contribution from other sources, and trap efficiency.

To develop the delivery rate curve, a non-linear regression equation was computed by the "least squares" method, using the size of the watershed and the measured delivery rate as the two variates.(11) The exponential regression curves with "X", or size of watershed, in square miles as the abcissa and "Y", or calculated delivery rate in percent of gross erosion as the ordinate, are shown. (See Fig. 2.) The non-linear standard error of estimate is 5.06 percent. This is a measure of the variation or scatter about the line of regression.

Three standard errors of estimate, measured plus or minus from the arithmetic mean, include 99.7 percent of the cases. Therefore, 3 x 5.06 percent = approximately 15 percent as the safety factor required to insure that in 99.7 percent of the cases the delivery rate will not be underestimated.

MANER, in a study of sediment delivery rates in the Red Hills physiogrphic area of Oklahoma and Texas(8) found that relief and maximum length of watershed expressed as relief-length (R/L) ratio(10) in a simple curvilinear correlation, or as individual variables in a multiple curvilinear correlation offered a much closer correlation with sediment delivery rates than size of sediment-contributing area or several other factors tested in the study.

Data from 25 reservoir sedimentation surveys were used in developing the watershed sediment yield values for the study, which included the following six steps: (1) tabulation of existing reservoir sedimentation survey data after making adjustments for reservoir trap efficiency and size of net contributing area to determine total sediment yield per square mile of contributing area; (2) study of rates of gross erosion during the period of record; (3) computation of sediment delivery rates by use of data obtained in steps (1) and (2); (4) detailed studies of the watershed of each reservoir to assemble data on physical characteristics which were thought to influence downstream delivery of erosional material; (5) correlation of data obtained in step (4) with sediment delivery rates to develop a regression equation for estimating sediment delivery rates; and (6) determination of the characteristics of the independent variables used in the equation.

The delivery rate curve (Fig. 3) was developed by the following equation:

$$\text{Log DRe} = 2.94259 - .82362 \text{ Colog } (R/L) \text{ ratio}$$

where DRe = Estimated sediment delivery rate in percent of annual gross erosion

R/L = Relief-length ratio

The standard error of estimate for the above equation is $\pm$ 0.04041 log units and the coefficient of curvilinear correlation is 0.987, indicating a highly significant correlation between these two variables.

Although delivery rate curves have been developed only in these two physiographic areas within the Western Gulf states, they serve as a guide to estimate rates of sediment delivery in several additional physiographic areas. Proper adjustments must be made for significant differences in various environmental factors when using the curves in such areas.

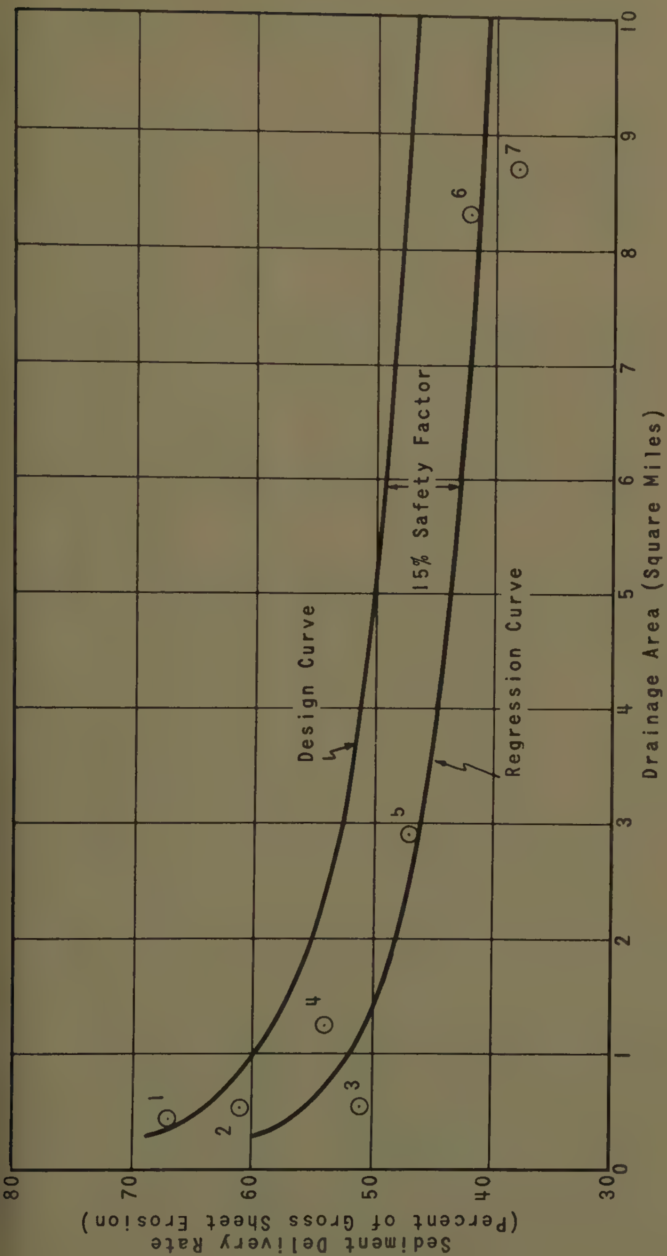

FIGURE 2

DELIVERY RATE CURVE

Calculated Sediment Delivery Rate (Percent of Gross Sheet Erosion)

VS.

Drainage Area (Square Miles)

For use ONLY in the Blackland Prairies Problem Are in Soil Conservation

U.S. DEPARTMENT OF AGRICULTURE, SOIL CONSERVATION SERVICE, FORT WORTH, TEXAS

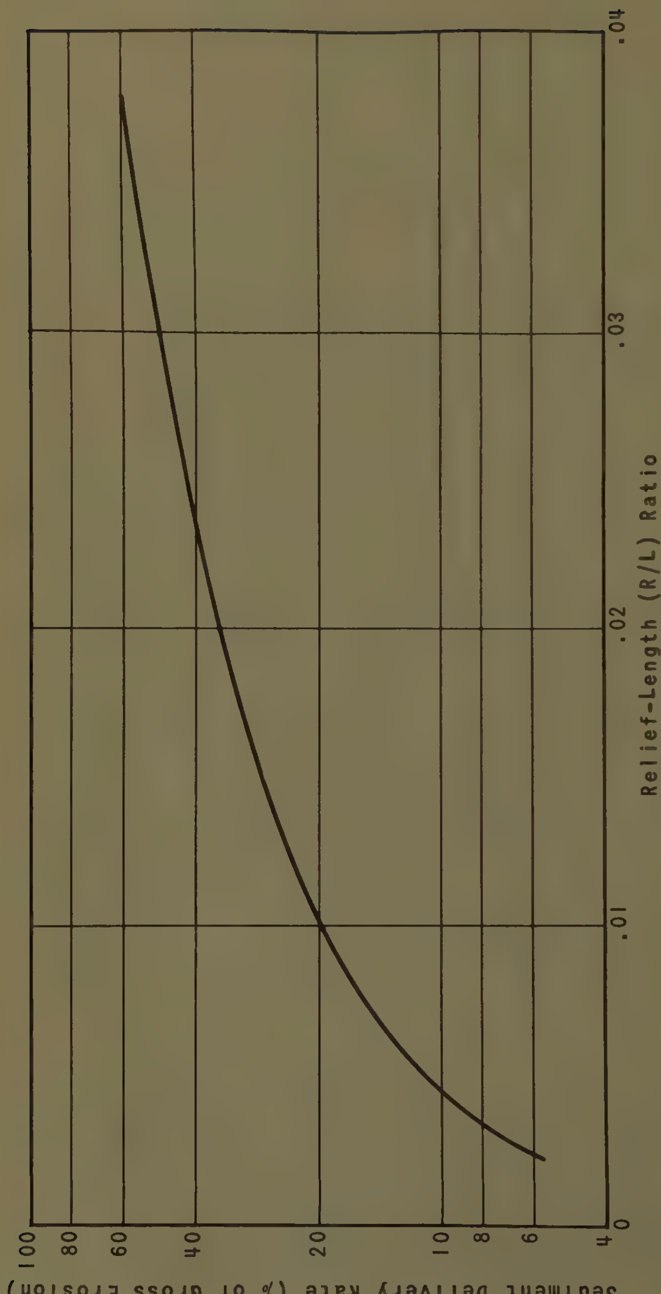

FIGURE 3

DELIVERY RATE CURVE

RED HILLS PHYSIOGRAPHIC AREA

Sediment Delivery Rate (% of Gross Erosion)

Related To Relief-Length (R/L) Ratio

U.S. DEPARTMENT OF AGRICULTURE, SOIL CONSERVATION SERVICE, FORT WORTH, TEXAS

Summary of Sedimentation Data for Design

After a study of the watershed above a proposed floodwater retarding structure has been completed, including computation of annual gross erosion for both present and future conditions and application of the proper sediment delivery rate to this eroded material, the information is summarized for design purposes.

Gross erosion, by sources, is totaled for both present conditions and the amount expected to occur after proper land treatment measures have been applied. The land treatment measures will be most effective in reducing sheet erosion. The effect of additional measures, such as special stabilization treatment for gully erosion is also considered in estimating future erosion from gullies.

The delivery rate is then applied to both present and future conditions. The prevailing rate of sediment yield is used for the estimated installation period for regular land treatment measures, normally 10 years. The rate of sediment yield for the remainder of the 50-year period is a realistic estimate based on the expected amount and effectiveness of proposed land treatment and other sediment control measures applied on the watershed. For example, it might be determined that 80 percent of the needed land treatment measures will be applied at the end of the 10-year period and maintained at 75 percent effectiveness.

The total sediment yield to the proposed structure for the 50-year period is then adjusted for trap efficiency. Available data indicates that the trap efficiency of floodwater retarding structures of the type being designed by the Soil Conservation Service in the Western Gulf states will be 85 to 95 percent. Cooperative studies by the U. S. Geological Survey and the Soil Conservation Service are now under way to determine the trap efficiency of selected floodwater retarding structures.

The next step is to make an adjustment for the difference in the dry weight of soil in place and sediment in the reservoir. Deposits in the detention pool or in dry pools will be subjected to aeration and will compact to a lesser volume than deposits in a permanently submerged sediment pool. In computing the volume of sediment in dry pools and in the floodwater retarding pools, ultimate compaction of aerated deposits is assumed. Computation of volume of sediments below the normal pool level will be based on ultimate compaction under submerged conditions.

Preliminary results of volume weight analysis of samples in Texas, Oklahoma and Arkansas indicate that the ratio of sediment volume in a permanently submerged sediment pool to soil in place will average 1.3:1.0.

The final step in summarizing the sediment yield data for design purposes is to allocate the total sediment yield to various pools of the retarding structure. The distribution of sediment in the reservoir is dependent on the nature of the sediment, the topography and shape of the reservoir, nature of the approach channel above it and the time required to empty the retarding pool. Allocation of capacity for sediment accumulation in the reservoir is made in both the sediment pool and the overlying retarding pool. In the Western Gulf States the proportion of the total sediment yield deposited in the retarding pool area has been between 10 and 30 percent.

In the following table a comparison between estimated and measured sedimentation rates in Soil Conservation Service floodwater retarding structures is shown:

Floodwater Retarding Structure	Date Constructed	Estimated Rate (Ac.ft./sq.mi./yr)	Date Surveyed	Measured Rate (Ac.ft./sq.mi./yr)
Sandstone Cr. No. 3 - Okla.	1949	1.10	1956	1.46
Sandstone Cr. No. 5 - Okla.	1949	1.55	1957	1.51
Sandstone Cr. No. 6 - Okla.	1949	1.40	1957	1.98
Sandstone Cr. No. 14 - Okla.	1949	2.23	1956	2.06
Six Mile Cr. No. 6 - Ark.	1954	1.05	1956	2.09
Green Cr. No. 1 - Tex.	1954	1.07	1957	3.22
Double Cr. No. 5 - Okla.	1954	0.45	1957	0.51

Measured rates of sediment accumulation in the Sandstone Creek structures checked closely with estimated rates particularly in the case of structures Numbers 5 and 14. The measured rate in Double Creek, site 5 also compared favorably with the estimated rate. The records in Sandstone Creek are considered more reliable than those in the other watersheds listed, since they encompass longer periods of time. In addition, rainfall averaged about normal during the period of record with no unusual storms occurring. During the short term record of Site No. 1, Green Creek one 25-year frequency and one 100-year frequency storm occurred. In Six Mile Creek, also a short term record, two storms exceeding a 5-year frequency occurred, both at times when antecedant rainfall had been significant.

These unusual storm events occurring during such short periods of record undoubtedly account for the high rate of sediment accumulation as compared to the estimated rate. It is believed that the average annual rate of sediment accumulation in these structures will closely approximate the estimated rate over a long period of time. Periodic resurveys will be made to determine rates of sedimentation and capacity loss in the above listed and other Soil Conservation Service floodwater retarding structures.

Effects of Watershed Protection Measures on Major Downstream Works

The watershed protection projects of the Soil Conservation Service include the treatment of the land with a good soil and water conservation program. In addition, they include such structural works of improvement as floodwater retarding structures, sediment control structures, floodways and floodwater diversions.

The land treatment phase of the program includes such measures as pasture, range and woodland improvement practices, improvement of soil fertility, terracing and contour tillage. It usually requires five or more years, depending on the extent of the job, to get the conservation work done.

The effects of conservation practices and watershed treatment upon the sediment yield of watersheds have been determined in numerous localities in the United States by the Soil Conservation Service.(5) Much of the data has been brought together in a recent publication.(6) A few of the outstanding results are as follows:

The rate of sediment yield to Lake Waco in Texas was reduced by 38 percent as a result of land use adjustments and conservation measures on

less than one-half of its 1,666 square-mile drainage area; conservation practices on 35 percent of the 62.3 square-mile drainage area of High Point Reservoir in North Carolina resulted in a 24 percent reduction in sediment yield to the reservoir; silting of Lake Issaqueena in South Carolina was reduced 53 percent following intensification of erosion control practices on about 20 percent of the 14 square-mile drainage area; the rate of sediment production of the 2.26 square-mile Jones Creek watershed in western Iowa was reduced by an estimated 98 percent as a result of multiple practices for erosion control and a highly effective sediment retention structure at the lower end of the watershed.

Although there is particular need for more analysis and research on the effects of conservation measures and structural treatment upon sediment production from watersheds, there appears to be ample evidence that such measures and treatment are not only of tremendous on-site significance but that they also contribute in a large degree to the reduction of major sediment damages downstream.

REFERENCES

1. Barnes, L. H. and Maner, S. B., Suggested Criteria for Estimating Gross Sheet Erosion and Sediment Delivery Rates in the Blackland Prairies. USDA, Soil Cons. Serv., Ft. Worth, Texas, Feb. 1953.
2. Brune, G. M., Maner, S. B., Renfro, G. W., and Ogle, J. A., Rates of Sediment Production in the Western Gulf States, USDA, SCS-TP-117, March 1956.
3. Eakin, H. M., Silting of Reservoirs, U. S. Dept. of Agriculture Tech. Bull. 524 (Revised by C. B. Brown), 166 pp. illus. 1939.
4. Glymph, L. M., Studies of Sediment Yields from Watersheds. Proceedings, Tenth General Assembly of the International Union of Geodesy and Geophysics, Rome, Italy, Sept. 15-19, 1954.
5. Glymph, L. M., Effects of Conservation Measures and Watershed Treatments Upon Flood Runoff and Sediment Production, USDA, SCS, Lincoln, Nebraska, March 1952.
6. Glymph, L. M. Relation of Sedimentation to Accelerated Erosion in the Missouri River Basin, U. S. Dept. of Agri., SCS, Lincoln, Nebr. SCS-TP-102, 1951.
7. Gottschalk, L. C., Measurement of Sedimentation in Small Reservoirs, Proceedings, ASCE, Hydraulics Div., Jan. 1951.
8. Maner, S. B., Factors Affecting Sediment Delivery Rates in the Red Hills Physiographic Area, USDA, SCS, Ft. Worth, Texas. (In press)
9. Musgrave, G. W., The Quantitative Evaluation of Factors in Water Erosion—A First Approximation, Journal of Soil and Water Conservation, Vol. 2, No. 3, pp 133-138, July 1947.
10. Schumm, Stanley A., The Relation of Drainage Basin Relief to Sediment Loss, U. S. Dept. of Interior, Geological Survey, Denver, Colo., 1954.
11. Snedecor, George W., Statistical Methods, 485 pp. fourth edition, The Iowa State College Press, Ames, Iowa, 1946.

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SOME EXPERIMENTS WITH EMERGENCY SIPHON SPILLWAYS[a]

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(Proc. Paper 1807)

SYNOPSIS

Two model designs of low head siphon spillways were tested. One is patterned after a standard design used for many years as an emergency-type of structure for the protection of canals. The second is a proposed design for an improved structure to perform the same function.

Operational characteristics and peculiarities of each are discussed and supporting data and pictures are included. The results of one prototype test of a standard design are also given. In the conclusions, studies of both designs are reviewed and future tests are outlined.

INTRODUCTION

The Bureau of Reclamation is conducting a program of research into the design and operation of low head siphon spillways. The primary function of the Bureau siphon spillway is to provide automatic emergency protection of canals, tunnels, and related structures. In performing its service, installations consist of single as well as multibarrel units, some with staggered crest levels, with fixed or adjustable crests, and with various air venting devices. The siphons to be discussed here are of the single-barrel, fixed-crest type.

Numerous siphons have been build along irrigation canals to prevent an excessive rise of the water level above the normal operating level caused by incoming storm water or the accidental closing of a control gate. Storm water usually enters a canal at such a rate that the rise in water surface is not rapid; hence, the siphon may operate for some time as a weir before priming, if, indeed, it ever primes. The rate of rise immediately upstream

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a. Presented at the October 1957 Convention in New York, N. Y.

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from a closing radial gate in a canal is somewhat more accelerated, particularly if the closure is accomplished in a short time; therefore, the siphon hardly operates as a weir before the excess head is sufficient to bring about a prime.

Siphons are also used in the forebay upstream from the entrance to a power penstock. An emergency closure of the turbine wicket gates, following powerplant failure, could cause the forebay level to rise very rapidly if the storage area were small. The siphon would then be required to prime immediately to avoid overtopping of the feeder canal banks or damage to the tunnel outlet structure if the forebay were tunnel fed.

Many designs of siphons have been used in the United States, each one usually being an improvement over its predecessor. Because of the cost of model and prototype studies, however, not many comprehensive analyses of the unsolved problems involved have been made. Difficulty in scaling up the results of model data, and the attendant lack of confidence in doing so, may be partially responsible for the significant lag in siphon technology in this country. At the time the studies reported herein were begun, a search of foreign literature revealed that Italy and France, and to a certain extent India, were more advanced than the United States in siphon development. The first two countries, for instance, are today using large siphon spillways for the close control of reservoir levels. These structures make use of so-called partialization boxes to accomplish an automatic proportioning of air and water in the barrel; the siphon is thus able to adjust for a rising or falling water surface while operating at a partial prime.

It had long been noted that some of the Bureau siphon spillways were not operating as desired. Some operated much above, and others below, the design capacities, the differences estimated to be as high as 30 to 40 percent. Others failed to prime even when the head over the crest exceeded the depth at the crown. In an outstanding instance, one siphon in a pair of identical structures built by the same contractor failed to prime while the other functioned satisfactorily.

The above facts led to the inception of a research program to learn first, how the standard performed, then why it failed occasionally to perform properly, and later, how to develop a new design for an improved siphon spillway.

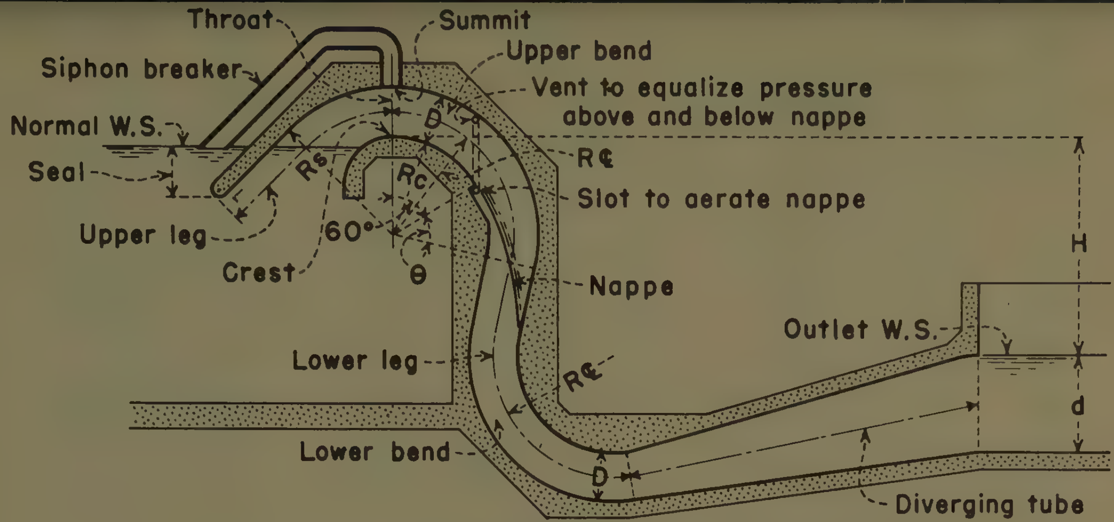
The material which follows lays greater stress on tests of a standard low head siphon spillway design than on the proposed design of a seemingly better structure. This disproportion in the discussion which follows can be understood when it is realized that more comprehensive tests were made on the standard design in efforts to gain a firm understanding of basic siphon principles; hence, a greater preponderance of data is available, and tests of the proposed design are not yet complete.

Standard Design of a Low Head Siphon Spillway

Design Considerations

A general dimensionless drawing of the standard low head siphon spillway is shown in Figure 1. The nomenclature of its various parts as given will be used throughout this paper. Losses in the crest region will be minimized if at a given section the pressure and velocity distributions correspond closely to those associated with vortex flow. After determining a crest elevation and the head available, and selecting preliminary values for the center line

FIGURE 1



TYPICAL LOW-HEAD SIPHON SPILLWAY

(Maximum head = atmospheric pressure equivalent at site)

DESIGN DATA

$$q = CD\sqrt{2gH} \leqq Rc\,\sqrt{0.7h\,(2g)} \times \log_e \frac{Rs}{Rc} = (\text{Vortex flow} \backsim \text{Max. } q)$$

q = Cubic ft. per sec. per ft. of crest width	d = Depth of water at outlet in ft.
Rc = Radius of crest at throat in ft.	h = Atmospheric pressure at site in ft. of water
R℄ = Radius of barrel center line in ft.	H = Available head
Rs = Radius of summit at throat in ft.	C = Discharge coefficient based on d/D and $R℄/D$
D = Throat height in ft.	g = 32.16

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radius $R_{℄}$ and the throat size D, one may solve for the maximum unit-width discharge in the vortex flow formula[1]

$$q = R_c \sqrt{0.7h(2g)}\ \mathrm{Log}_e \frac{R_s}{R_c} \tag{1}$$

where

q = discharge in second-feet for a unit width
R_c = radius of the crest in feet
R_s = radius of the summit in feet
h = atmospheric pressure at site in feet of water

In practice, a minimum D = 2 feet is recommended, and by using the ratio $\frac{R_{℄}}{D} = 2.5$, the coefficient of discharge will be near the maximum obtainable. From field information consisting of topographical and hydrological data, D and H may be approximated and a value of the coefficient of discharge obtained from Figure 2. Substituting, the unit q may be solved for in

$$q = CD\sqrt{2gH} \tag{2}$$

where

H = head in feet from crest to tail water elevations

The unit discharge in (2) must not exceed that in (1). If it does, the siphon should be reproportioned and (1) and (2) solved again. The width of the siphon can then be determined by dividing the total discharge by the unit discharge. For the purpose of setting the crest elevation, an allowed priming head must be assumed which satisfies the normal water surface and freeboard elevations. The designer's judgment is an important factor here.

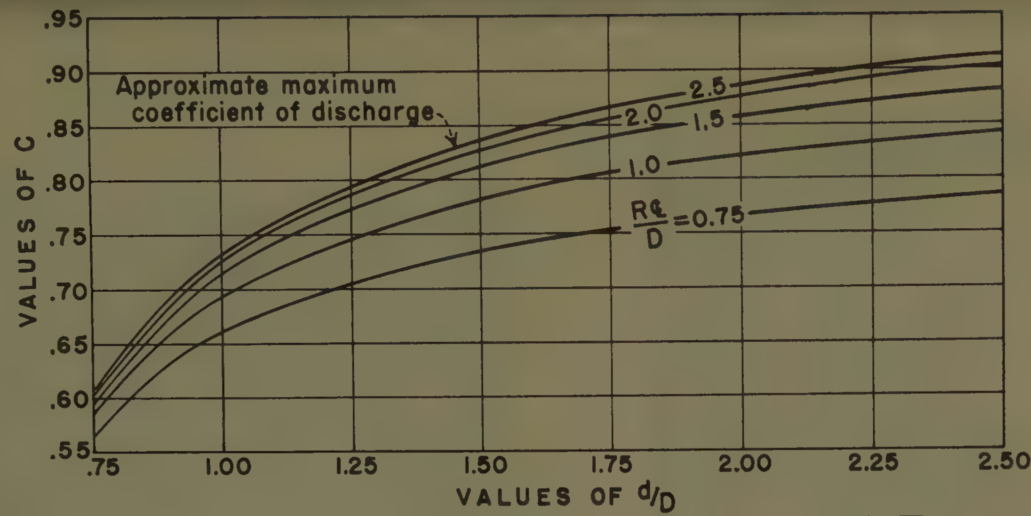
The size of the siphon breaker pipe is arbitrarily assumed to have an area 1/24 that of the siphon throat area, or that of the nearest standard pipe since it is usually convenient to use pipe sections and fittings for the breaker. The inlet elevation of the breaker should be set at, or just below, the normal water surface elevation.

Laboratory Test Facilities

The test setup is shown in Figure 3. Part A of this figure shows the forebay, the entrances to two siphon spillways, and water stage recorder box and float well. The net forebay area is 144 square feet. Two siphons were installed; one on the left, shown in elevation in Figure 3B, was fixed in position, and the one on the right was adjustable. The adjustable could be raised or lowered by a hydraulic jack to select the siphon desired for operation. Flow conditions of two designs thus could be compared readily by alternate operation. Each siphon discharges into a separate tailbox equipped with a 3-foot suppressed rectangular weir.

1. Design Data of Typical Low-head Siphon Spillways, Reclamation Instructions, Volume X, Part 2, Design Supplement 3. Coefficient of 0.7 under the radical chosen as a safety factor.

FIGURE 2



VALUES OF C FOR USE IN EQUATION $q = CD\sqrt{2gH}$

Curves based on following assumed losses:

Entrance..................0.20 hv (Siphon throat)
Friction (h_f).............0.25 hv (Siphon throat)
Bends...($R℄/D = 2.5$)....0.42 hv (Siphon throat)
Outlet { Diverging........0.20 Δhv (hv at throat – hv at outlet)
Outlet { Converging......0.10 Δhv (hv at outlet – hv at throat)

TYPICAL LOW-HEAD SIPHON SPILLWAY
COEFFICIENTS OF DISCHARGE

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Fig. 3-A. Model Forebay, Showing Entrances to Two Siphon Spillways and Water Stage Recorder Box.

Fig. 3-B. View of Standard Siphon Spillway, with Piezometer Tubes and Manometer Board.

Water was supplied to the model from a large volume channel by a centrifugal pump. The rates of flow were measured through 6-, 8-, and 12-inch venturi meters which had been calibrated volumetrically.

The forebay water surface was measured with a hook gage during constant siphon discharges. When a siphon was operating at less than full-primed capacity, the fluctuating level was measured and recorded with a modified Stevens A-35 water stage recorder. The modification consisted of an electric motor and speed reducer which drove the recorder paper at a speed of 1/10 inch per second. The pen travel was on a 1:1 ratio with the variation of water level. An event marker with remote manual control enabled the operator to record the occurrence of any significant operational characteristics.

General Types of Siphon Operation

In general, a siphon spillway can operate in any one of three ways. First, the excess flow to be siphoned away may be low enough to prevent continuous operation. The forebay level will rise slowly to an elevation at which the siphon can prime, the forebay drops as the siphon operates a short time fully primed, and then as the siphon breaker pipe is uncovered, the prime is lost and flow over the crest stops. This is the type of operation one may expect on a small canal where the siphon capacity is a high percentage of the canal capacity. A typical stage versus time curve is shown in Figure 4A.

Second, the excess flow is great enough to prime the siphon and sufficient to maintain a flow through it at a partial prime. In this case, the inflow keeps the siphon breaker pipe covered, or lets an intermittent slug of air enter in a quantity too low to fully break the prime, and the water level within the barrel remains below the summit. The siphon will continue to operate in this stage until there is a change of inflow. An increase will bring the siphon closer to a fully primed state; however, a decrease may cause a complete break in prime and temporary cessation of flow. The stage versus time curve appears as in Figure 4B.

In the third type of operation, the excess inflow must reach the siphon in a comparatively short period of time at a rate which causes it to prime immediately and to remain primed. The siphon barrel will then be full from inlet to outlet. The forebay water surface elevation can be lower than the summit elevation for this type to exist. Pressures in the throat section reach maximum negative values when the barrel is flowing full and the difference in elevation between crest and forebay water surface is a steady maximum. Any increase in excess flow decreases the negative pressures at the throat. This type of operation is most apt to occur during a rejection of flow at a powerplant. Figure 4C shows the stage versus time curve.

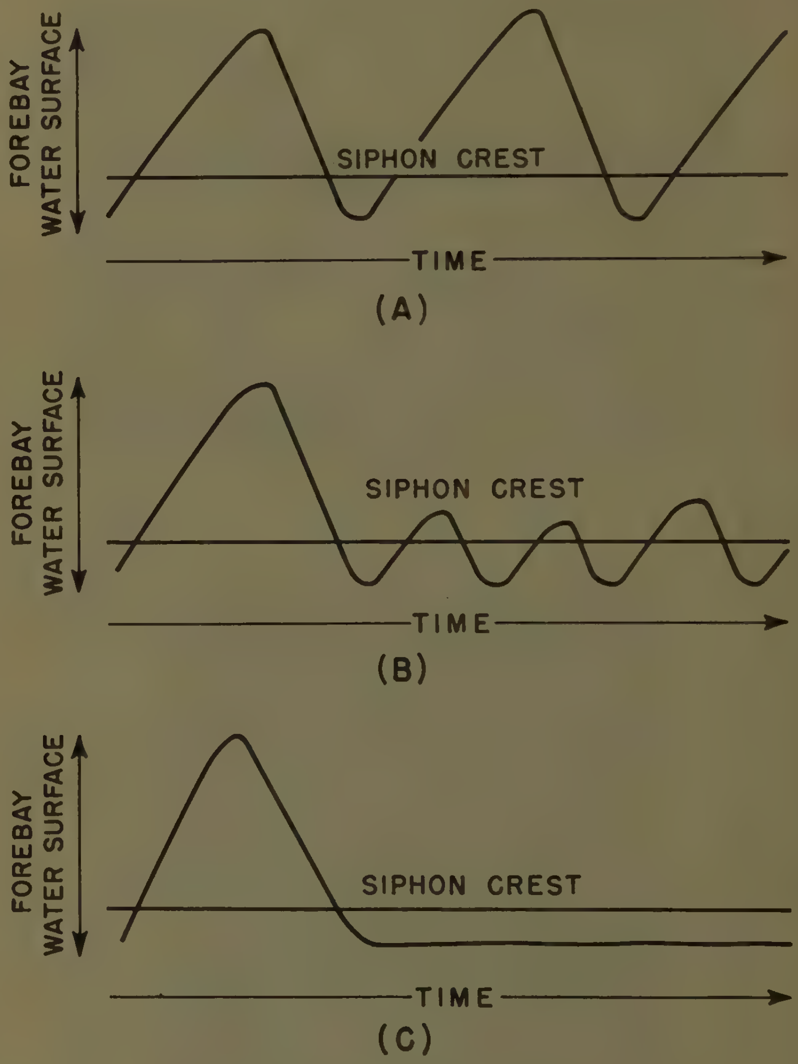
Of course, there are infinite variations of the above stages of operation, but for purposes of this study, the three general ones defined were adequate.

Operating Characteristics

The standard siphon spillway design chosen for study in the laboratory was of the type constructed on the Mohawk Canal, Gila Project, in Arizona. Prototype dimensions were reduced on a 1:4 scale. The resulting siphon is dimensioned in Figure 5A. The locations of 50 piezometers utilized in obtaining pressure distribution patterns are shown in Figure 5B.

In Figure 5A, when the forebay inflow was sufficient to top the crest, the overflowing sheet followed the profile to the deflector lip. When the rate of

FIGURE 4



SIPHON SPILLWAY STUDIES

TYPES OF SIPHON SPILLWAY OPERATION

WATER STAGE VERSUS TIME

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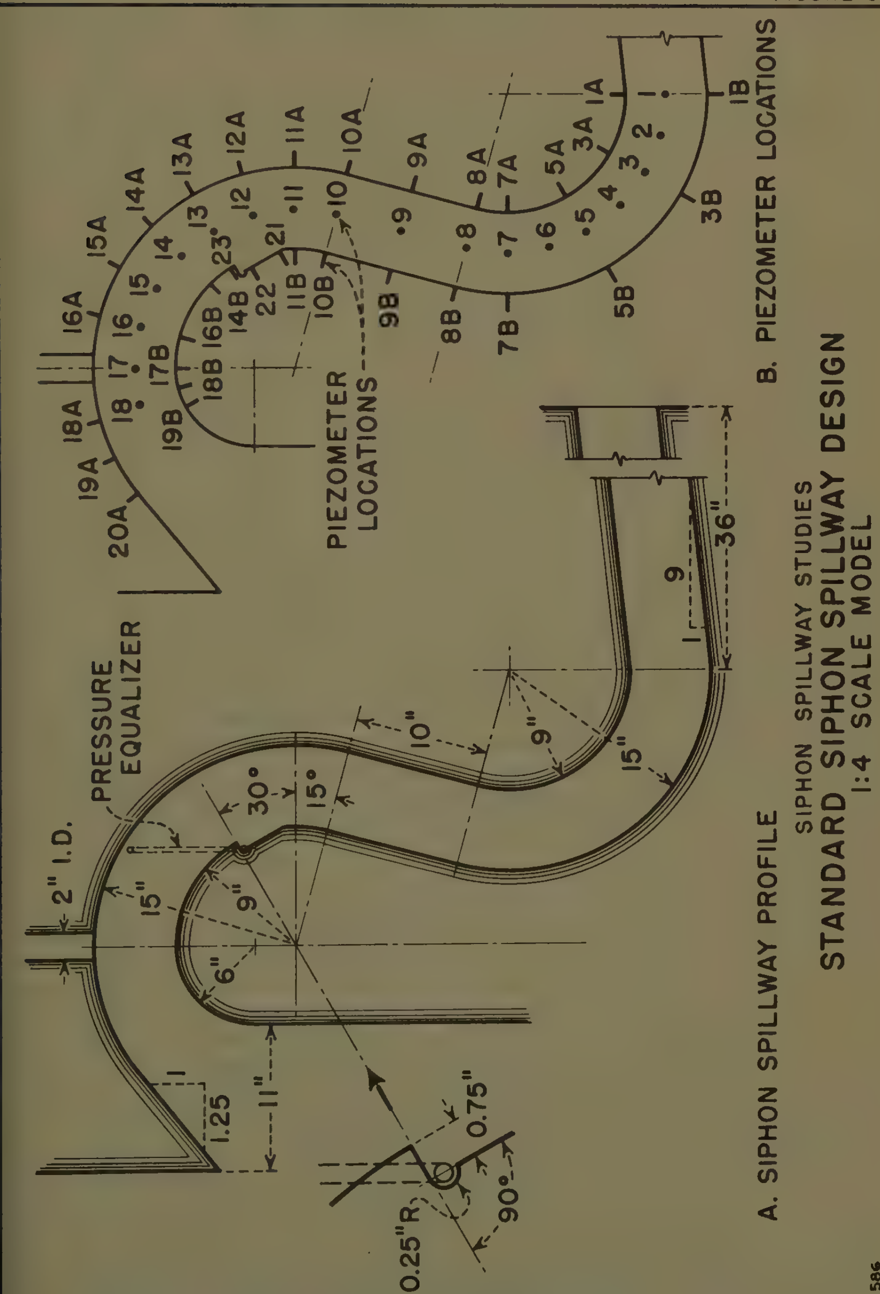
FIGURE 5



A. SIPHON SPILLWAY PROFILE

B. PIEZOMETER LOCATIONS

SIPHON SPILLWAY STUDIES

STANDARD SIPHON SPILLWAY DESIGN

1:4 SCALE MODEL

flow was still a very small amount, enough energy was available to allow an unbroken jet to leave the lip in a direction tangent to it, cross the barrel, and strike the opposite boundary. Along that line of impact, the jet broke into flows in all directions, though the principal one was downward. An effective seal was accomplished at the point of impact which divided the barrel into two parts, an upper air cavity and a lower one. As the head in the forebay rose, air in the upper cavity was forced through the pressure equalizer to the underside of the nappe. Air flows rather freely to the lower air cavity because of an air demand at this point created by the jet of water. In fact, a better equalization would result if it were possible to install a larger diameter pipe. The larger it is, the quicker the siphon will prime.

At this stage of operation, any but a very slow rate of rise in the forebay produced a positive pressure in the lower air cavity. The pressure continued to rise only until it was great enough to break through momentarily the free jet of water, thereby restoring the equality of pressures in both cavities. When this happened, priming was further delayed.

Following impact of the jet of water against the opposite boundary most of the water fell immediately into the lower bend pool where water had sealed at the roof of the bend. The air in both cavities must be evacuated by means of turbulent mixing past this seal far enough to rise of its own accord to the free water surface at or near the exit of the outlet tube. Our experience with this design showed that considerable turbulence was required in the lower bend pool to begin to move air out, and until that degree of turbulence did exist the forebay head continued to increase. While the reverse curvature of the lower leg aided materially in obtaining an adequate seal and separation of air cavities, it also caused water to move toward the lower bend with a direction component opposite that conducive to discharge through the siphon.

The design tested was the outgrowth of studies on vertical drop siphon spillways in which similar crest profiles fed a vertical conduit with a pool and sharp bend at the lower end. The amount of drop was merely varied to fit the particular site of the structure. Consequently, only by chance did the required discharge, operating head, crest profile, length of drop, and shape of lower bend combine hydraulically to operate correctly. The reverse curve and the deflector lip of the present design did much to improve the former design.

Very early in the investigations it became obvious that the meaning of "prime" must be positively defined. Many of the technical papers studied as a prelude to this work made reference to the time required for priming of a siphon, with no criterion established for determining what constituted a prime other than visual observation. Therefore, a primed siphon was defined as one in which the pressure at the crown or summit had reached a maximum negative value for the particular forebay inflow. So defining the term allowed us to measure with reasonable consistency the length of time required for a siphon to reach a degree of prime in keeping with the inflow. The negative pressure does not reach the maximum possible value until the forebay inflow reaches the capacity discharge of the siphon at a constant head.

The maximum discharge for the model was 5.70 cubic feet per second; a full prime was reached at 5.35 cubic feet per second. The coefficient of discharge, based on the elevation difference from forebay water surface to the outlet roof, was 0.84. This value is high compared to those measured on some prototype siphons of similar configuration, but the model was built under laboratory controlled standards and tested in an ideal setup, two factors conducive to a higher coefficient.

Table I

PIEZOMETER PRESSURES
Siphon Spillway Model--Standard Design
Scale 1:4
Q = 5.35 cfs

Piezometer	Pressure in: ft of water	Piezometer	Pressure in: ft of water	Piezometer	Pressure in ft of water
1	0.82	9	-0.85	15	-2.14
1A	-0.98	9A	-0.79	15A	-1.69
1B	1.49	9B	-0.91	16	-2.17
2	0.73	10	-1.13	16A	-1.90
3	0.69	10A	-0.61	16B	-3.34
3A	-0.81	10B	-1.89	17	-2.14
3B	0.77	11	-1.28	17B	-3.32
4	0.56	11A	-0.65	18	-1.96
5	0.35	11B	-2.42	18A	-1.45
5A	-1.16	12	-1.53	18B	-3.58
6	0.09	12A	-0.94	19A	-1.22
7	-0.16	13	-1.77	19B	-2.88
7A	-1.63	13A	-1.22	20A	-0.62
7B	0.26	14	-1.94	21	-2.05
8	-0.47	14A	-1.47	22	-2.90
8A	-1.27	14B	-2.89	23	-2.90
8B	-0.23				

Table 1 gives the piezometer pressures measured for a fully primed condition at points shown in Figure 5B. Along the roof of the model all pressures were negative, with reference to their respective elevations reaching a maximum value just downstream of the siphon breaker inlet. On the side center line, pressures were negative from the inlet to a point beyond the PC of the lower bend, and then increasingly positive through the lower bend. Pressures over the crest and down to the PC of the lower bend along the floor were higher negatively than those in the other piezometer sets. Maximum pressure occured at Piezometer 18B, about -3.6 feet of water.

Since the design was based on the assumption of vortex flow at the crest, it is of interest to learn whether vortex-type flow does exist. With a flow of 5.35 second-feet, physical dimensions of the model were used to compute the head as given in the equation for vortex flow,[2]

$$Q = \sqrt{2gh}\,\frac{R^2 - C^2}{2\sqrt{R_C}}\,2.3026b\left[\log(R + C) - \log(R - C)\right] \qquad (3)$$

where
- Q = discharge in second-feet
- R = radius of barrel centerline at the crest section
- c = height of barrel at the crest section
- b = width of the barrel

The value of h so obtained was

$$h = 1.87 \text{ feet}$$

The corresponding head measured in the model and adjusted for elevation

2. Addison, Herbert, "A Treatise on Applied Hydraulics," Third Edition, page 168.

differences, since the siphon was in a vertical instead of horizontal plane, was

$$h = 1.94 \text{ feet}$$

These two values are close enough to justify acceptance of the vortex flow assumption.

Figure 6 is a composite of two sets of basic operational curves, one of maximum and minimum crest heads for the initial cycle of operation, and the other of crest heads for those cycles succeeding the initial cycle. About 0.125 foot of head at a discharge of 0.1 second-foot was required to prime the siphon. For discharges to 1.75 second-feet, operation is characterized in Figure 4A. The crest was dry between cycles of operation and each cycle was therefore independent of the one which preceded it.

From 1.75 to 3 second-feet, operation was as shown in Figure 4B. The initial cycle resulted in a higher head to prime the siphon, but succeeding cycles operated over a reduced range of heads. Some varying depth of water was continually flowing over the crest. The closer the discharge was to 3 second-feet, the narrower became the range of operating heads.

About 3 second-feet, the maximum crest head on the initial cycle continued to rise. At about 4.5 second-feet, the maximum head equaled the barrel height at the crest section. For cycles following the first one there was no difference in maximum and minimum crest heads, and operation was as shown in Figure 4C. As the capacity of the siphon was approached, the head curve for cycles following the initial one rose rapidly toward the head curve for the initial cycle.

The curve of priming time versus discharge for the initial cycle of operation only is shown in Figure 7. The shape of the curve, particularly the reversal characteristic, can be explained as follows. Below 2 second-feet, the rate of forebay rise was not sufficiently high to cause a significant compression of air in the upper air cavity; however, over 2 second-feet, the rate of rise became high enough to cause the pressure in the cavity to be forced above atmospheric before air pumping in the lower leg began. The result was to delay priming. Also, up to 2 second-feet the lag between the start of air pumping and rise in forebay was not as serious as it apparently becomes above 2 second-feet. The two explanations offered here are closely related, of course.

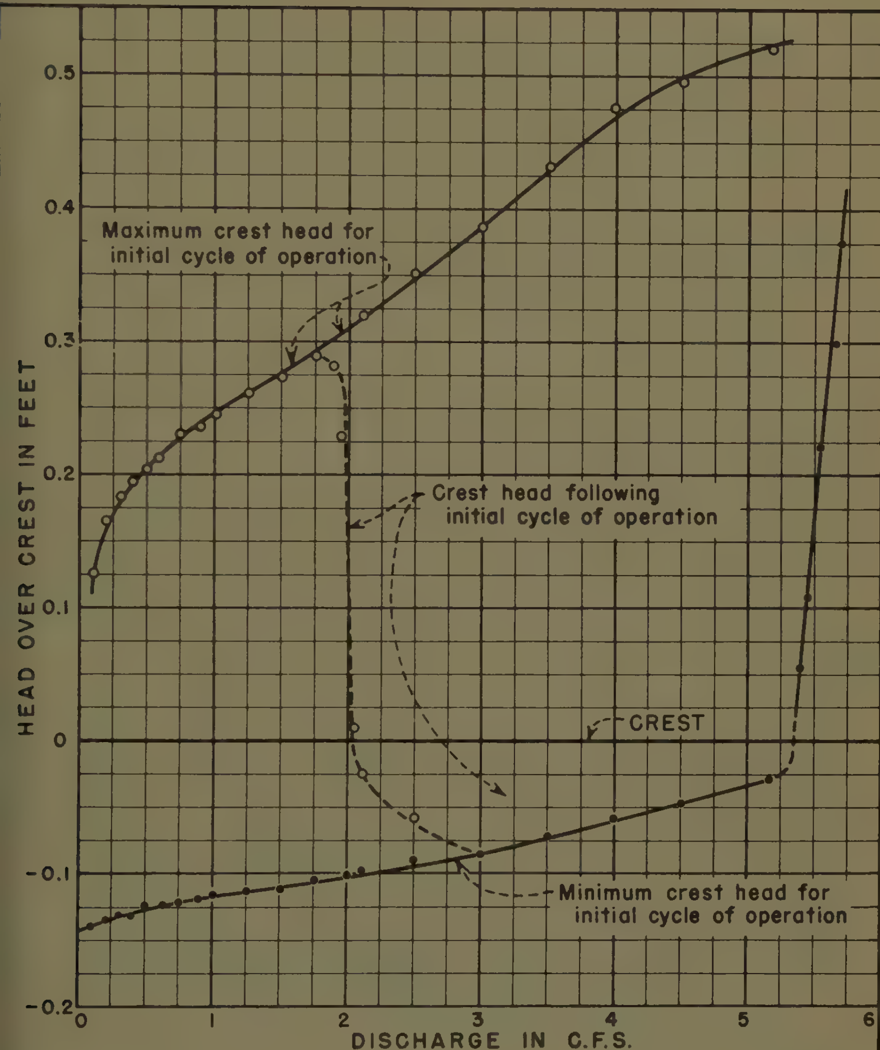
Prototype Test of Standard Design

The results of one prototype test were available for use in this investigation. A siphon spillway on the Mohawk Canal, Figure 8, closely resembling that modeled in the laboratory, was used to obtain pressure and velocity profile data. The barrel was of constant section, 2 feet by 4 feet. At five stations along the roof of the structure, pitometer outlets were utilized to obtain velocity profiles across the barrel.

Designed for a capacity of 150 second-feet, the structure passed a maximum of 136 second-feet. The head assumed to be required for a prime was 0.17 foot, but in the tests 0.46 foot proved to be necessary.

In order to show whether vortex flow prevailed within the siphon, Table 2 was prepared. Measured velocities were multiplied by corresponding radii to determine how nearly constant the products were. An average VR value was

FIGURE 6

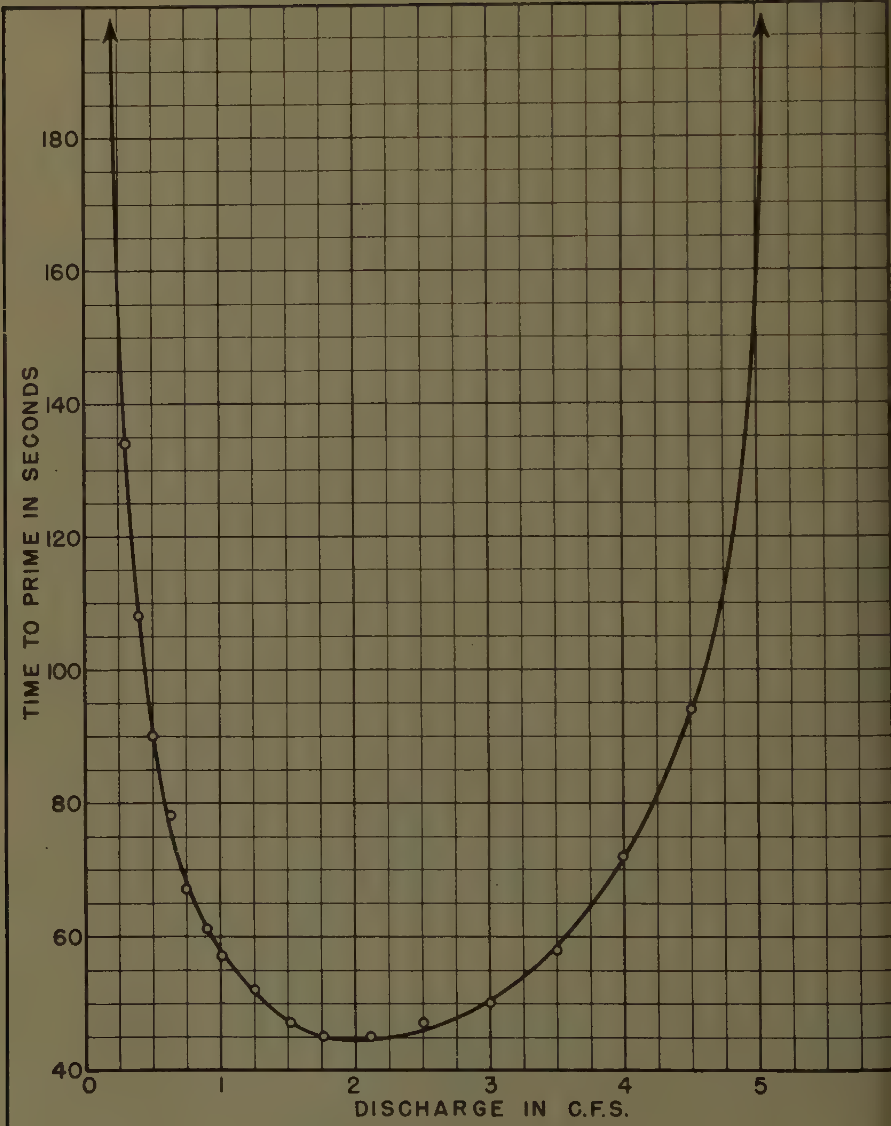


SIPHON SPILLWAY STUDIES

STANDARD SIPHON SPILLWAY DESIGN

CREST HEAD VERSUS DISCHARGE

FIGURE 7



SIPHON SPILLWAY STUDIES

STANDARD SIPHON SPILLWAY DESIGN

TIME TO PRIME VERSUS DISCHARGE,
INITIAL CYCLE OF OPERATION

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Table II

VARIATION OF VR = k WITH RADIUS IN PROTOTYPE SIPHON BARREL AT THE CREST SECTION

Radius	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0
Velocity	21.3	21.3	20.25	19.31	17.70	16.43	15.65	14.43	13.76	13.31	11.64
VR	63.9	68.2	68.8	69.5	67.3	65.7	65.7	63.5	63.3	63.9	58.2
Deviation from VR = 66.2*	-3.5%	+3.0	+3.9	+5.0	+1.7	-0.8	-0.8	-4.1	-4.4	-3.5	-12.1

*Average of all VR values, excluding those at boundaries.

computed neglecting boundary measurements, and the deviation from that value was noted. Boundary measurements were not included in the average because wall friction upset the measured velocities. Deviations from the average ran from +5.0 percent to -4.4 percent. This small range was considered adequate proof that near-vortex type flow exists, and it served to lend further credence to the model results.

Analysis of Model Operation, Standard Design

Several important conclusions were derived from studies of the standard design. Although this design assures positive separations of air volumes and seals from the atmosphere, the practice of bending back the lower leg to accomplish these objectives causes a serious delay in the removal of air from the barrel. This being true, the priming, though positive in action, is retarded, and the crest head required for priming is higher than desirable. Further, the built-in lag in priming produces greater head fluctuations than can be tolerated in most installations.

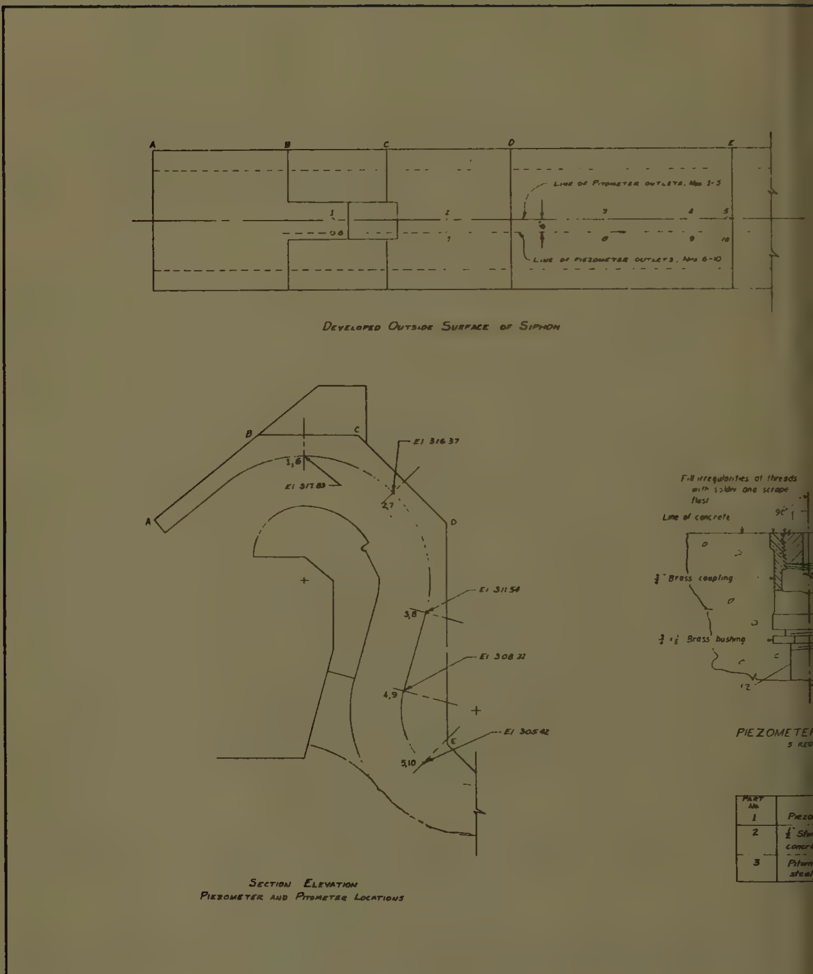
Numerous attempts were made to design special air-intake partialization devices for the model to replace the siphon breaker pipe. Our efforts were unsuccessful mainly because of the delay in the start of priming and the comparatively rapid rate of forebay rise. During the rise in the forebay water surface, the devices became submerged before sufficient subatmospheric pressure in the crown of the siphon could develop to draw in the needed proportion of air; and when the water surface dropped, siphon action was broken before the level reached the minimum stage desired. Thus, it was considered impossible to modify the model design to make use of a partialization device.

Limited success in prototype structures has been had with air slot partializers substituted for the siphon breaker pipe, but where some degree of partialization was attained the rates of forebay rise were very much slower than in our model tests. In one instance where the rate of forebay rise was very rapid, the air slots failed to accomplish partialization at all, due to flooding.

Although erosion downstream of the outlet of a siphon spillway was not studied with our models, in many locations the prime-break-prime type of intermittent operation, which is a characteristic in the lower third of the discharge range, could not be withstood.

The design tested is generally a satisfactory one when used where the rate of rise of the forebay water surface is low and erosion damage due to prime-break-prime cycling can be tolerated. Where the rate of rise is high, freeboard allowances in the forebay must be increased to accommodate the greater range of heads which must result.

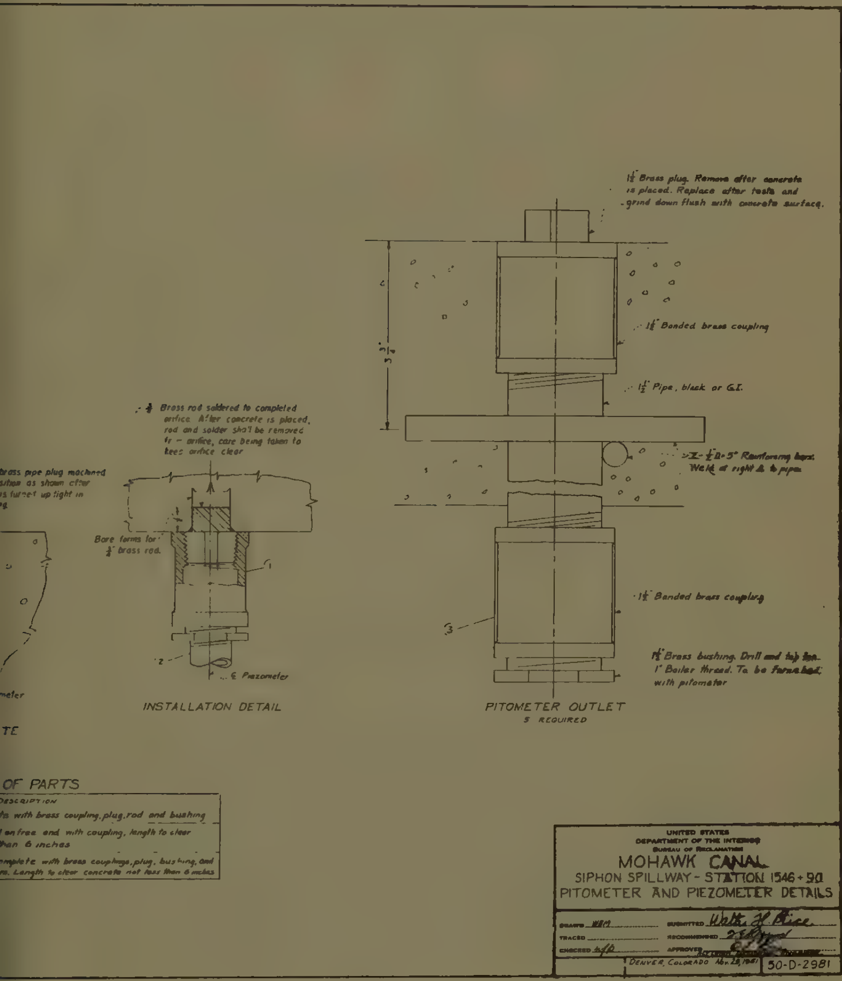
The model data yielded no positive clue as to why some prototype structures



Developed Outside Surface of Siphon

Section Elevation
Piezometer and Pitometer Locations

FIGURE 8



1½" Brass plug. Remove after concrete is placed. Replace after tests and grind down flush with concrete surface.
1½" Banded brass coupling
1½" Pipe, black or G.I.
¾" Brass rod soldered to completed orifice. After concrete is placed, rod and solder shall be removed from orifice, care being taken to keep orifice clear
Bore forms for ¾" brass rod.
1½" Banded brass coupling
1½" Brass bushing. Drill and tap for 1" Boiler thread. To be furnished with pitometer
INSTALLATION DETAIL
PITOMETER OUTLET
5 REQUIRED
OF PARTS
UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF RECLAMATION
MOHAWK CANAL
SIPHON SPILLWAY - STATION 1546+90
PITOMETER AND PIEZOMETER DETAILS
DENVER, COLORADO
50-D-2981

operated satisfactorily while others failed to do so. The answer to this problem must be sought in prototype tests on existing structures.

Proposed Design of a Low Head Siphon Spillway

Design Considerations

With the results of tests on the standard design of a siphon spillway in mind, a simplified siphon with more consistent and predictable characteristics was sought. Several objectives were established as guides for an improved design. One, a new siphon spillway should continue to make use of a vortex flow principle of earlier designs. Second, the crest head required for priming should be reduced. Closely related, third, was the desire to reduce the priming times for all heads. Fourth, if the second and third objectives could be attained, efforts should be made to fit the structure with some device for the partialization of air, thereby allowing the siphon to reach some degree of prime at low discharges without the necessity of becoming fully primed on the first cycle and then settling back on succeeding ones to a partialized state. It was hoped, too, that a forthcoming design would be less difficult to construct and could perhaps be simpler than the standard design.

Following library research, it was decided to pattern the new siphon after ones built and successfully operated in Europe. The design of the structure could not be proportioned much differently than the original, i.e., the approach hood and crest section must resemble the former design and if test results were to be easily compared, the total siphon head must be the same. But the conduit downstream of the crest as well as the lower bend were considerably changed in adapting the European designs to our use.

Figure 9 shows the proposed siphon spillway model design which evolved from these studies. The slope of the lower leg as well as the positions of the deflectors were arbitrarily chosen. Each deflector could be rotated from the flush position to that of a 45-degree projection. Divided arcs were installed to allow setting the deflection within 1 degree. To obtain a positive seal in the bucket, the downstream lip was constructed slightly higher than the downstream end of the sloping roof section.

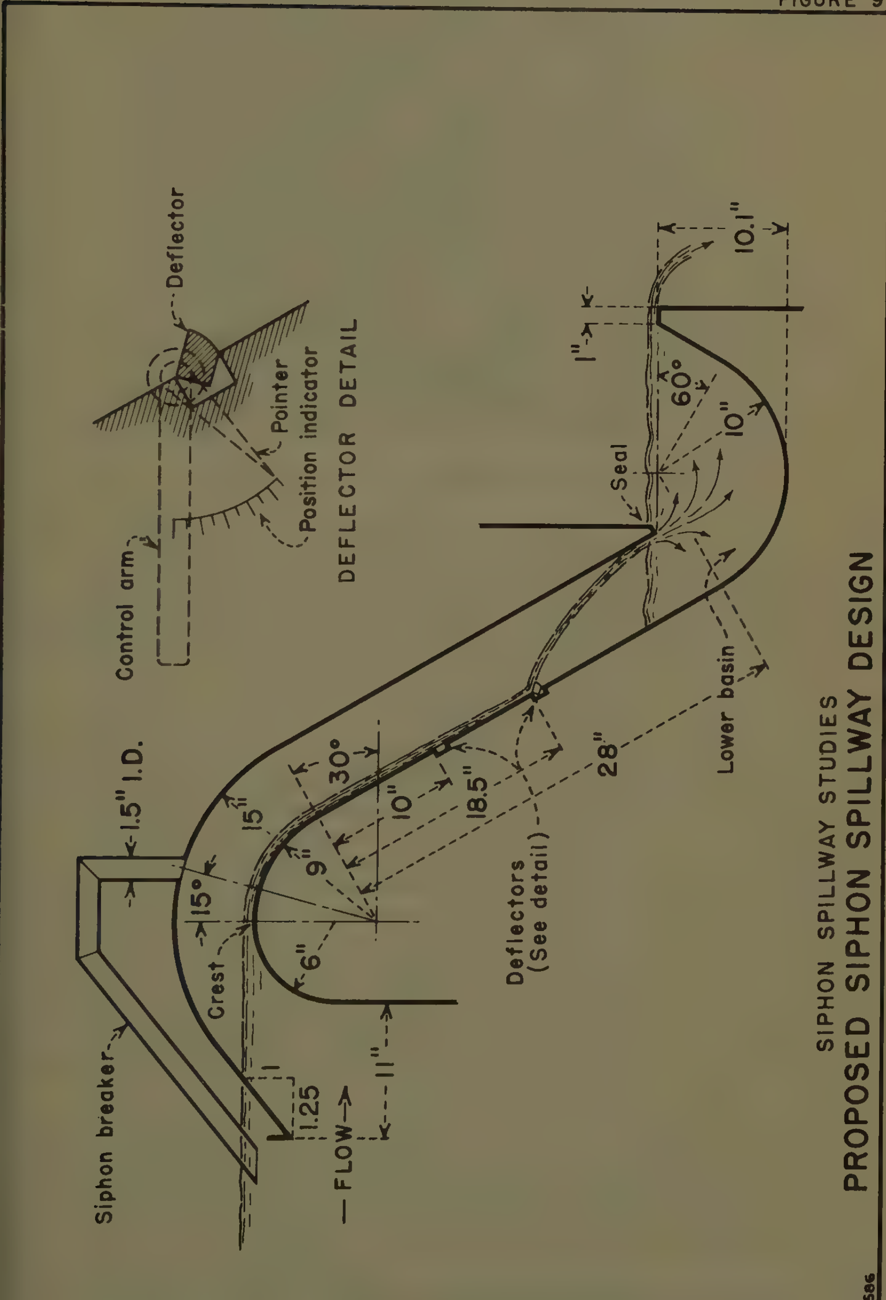
Since the maximum negative pressures near the crown of the standard design occurred some 15 degrees downstream from the crest, the outlet of the siphon breaker pipe was in the proposed design so positioned. The reason for choosing this location was that subatmospheric pressures probably begin developing in that location earlier than elsewhere.

No extensive sets of piezometers for the measurement of pressures were installed in this siphon. The pressure patterns and magnitude of pressures were sufficiently developed in tests of the standard design, and a repetition of this work was not thought to be necessary, despite the different configuration of the exit section of the structure.

The Model

The model of the proposed design was installed adjacent to that of the standard design, Figure 10. The vertical position of the siphon had to be adjustable so that only one siphon would operate at a given time. For this reason, the mounting plate, held between two greased plates, was allowed full freedom of movement from below to well above the position of the standard

FIGURE 9



SIPHON SPILLWAY STUDIES

PROPOSED SIPHON SPILLWAY DESIGN

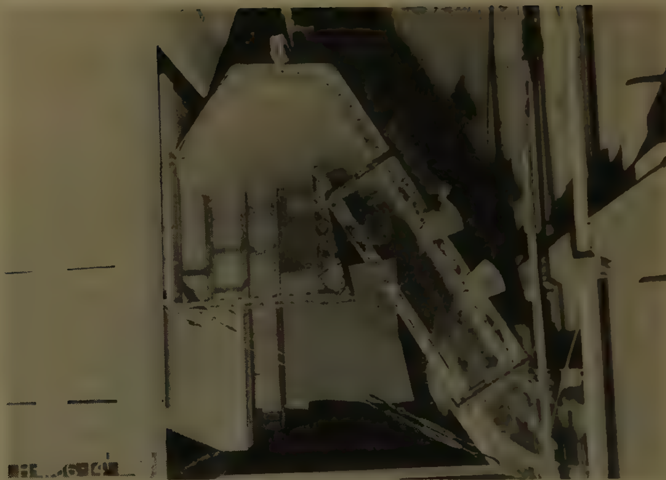


Fig. 10. Model of Proposed Siphon Spillway Design.

siphon. A hydraulic jack was used to change elevations of the model. A tail-box was built to accommodate the discharge over a range of positions.

Anticipating the need for measurement of the rate of air demand, the siphon breaker pipe was provided with an orifice plate section and accompanying piezometers. The position of the orifice will allow use of a partialization device upstream without further modification.

Operating Characteristics

Referring again to Figure 9, when water crossed the crest, the overflowing sheet accelerated down the slope toward one of the projecting deflectors. The direction of flow was then altered to throw the water in an unbroken sheet across the conduit to the roof of the descending leg. The position in which it initially struck the roof was dependent, of course, on the deflector angle. Though the jet was broken into flow components in all directions at the line of impact, the principal direction remained downward into the bucket pool. Impact with the roof and turbulence in the pool served to mix air and water well.

With the bucket full of water, an air cavity from forebay water surface under the hood to the bucket pool had to be evacuated before a full prime could be realized. If the elevation as well as angular position of the deflector were correctly chosen, a jet of water struck the pool very near the lowest edge of the roof. The turbulence thus created and the forward component of velocity combined to produce immediate air pumping.

Two deflectors were built in the model to enable evaluating the relative merits of each position. Though test results showed the lower position more effective than the upper one, it would be unwise to exclude the possibility that a third undetermined position might be better than either of the first two.

Results discussed here are limited to those obtained from the lower deflector.

The proposed design had a maximum capacity of 4.35 second-feet; a full prime was reached at 3.80 second-feet. The coefficient of discharge, based

on the elevation difference between forebay water surface and the downstream lip of the bucket, was 0.62. So computing the coefficient does not give recognition to the height the discharging water rises above the lip of the bucket, but even if the total head were reduced by that increment, the rise in C_d would not bring the value above about 0.69.

Thus, the coefficient of discharge for the proposed design is much below that of 0.84 for the standard design. It can be assumed that part of the reduction was caused by back pressure exerted by the turbulent head of water in and above the bucket. As will be seen in the characteristics that follow the reduced C_d can be tolerated, or compensated for by a longer crest length, when considered with operational improvements.

Figure 11 is the companion set of operational curves to Figure 6 on the standard siphon. About 0.035 foot of head at a discharge of 0.1 second-foot was required to prime the siphon. Up to 1.15 second-feet, operation is shown in Figure 4A. Each cycle was independent of preceding ones and had no influence on those which followed. The crest was dry therefore in the time interval between cycles.

From about 1.15 to 1.60 second-feet, the operation is as shown in Figure 4B. The crest head rose to a higher value on the initial cycle than on any following cycles. Water never quite ceased to flow over the crest. The closer the discharge was to 1.6 second-feet, the narrower the range of operating heads became after the initial cycle.

Above 1.6 second-feet, the maximum crest head on the initial cycle continued to rise, but for cycles following there was no difference in maximum and minimum heads. As the discharge approached 4.25 second-feet, the crest heads for all cycles became nearly the same. Between 4.25 and 4.35 second-feet, the crest head increased from 0.425 to 0.5 foot and the siphon capacity was reached. Operation above 1.6 second-feet is as shown in Figure 4C.

The curve of priming time versus discharge for the initial cycle of operation with the deflector set at 45° is shown in Figure 12. The displacement which took place about 1.15 second-feet can be explained as follows. At all discharges above this value, the rate of flow over the crest during priming was sufficient for a very short period of time to break the water seal at the low point of the roof. In other words, the turbulence at that location reached such a magnitude as to depress the solid water surface below the sealing edge, allowing a sudden slug of air into the air cavity of the barrel. The resulting momentary reduction of negative pressure in the cavity caused a delay in priming while the invading air increment was pumped out and the priming process was resumed.

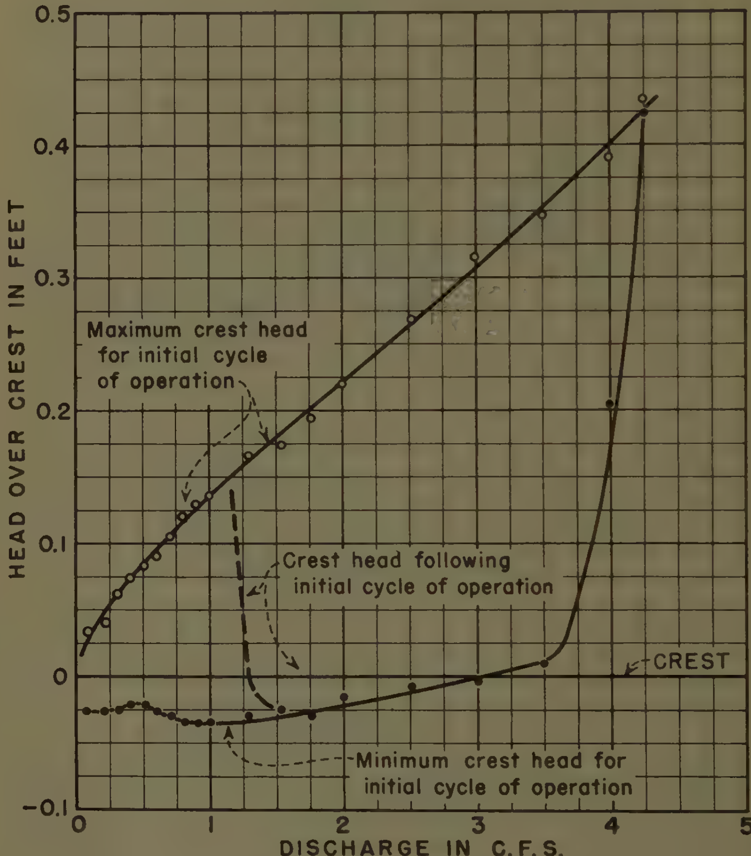
In the low range of discharges, up to about 0.5 second-foot, the crest head required for priming increased as the deflector angle decreased. If the angle were less than 20 degrees, priming became erratic and unreliable, the siphon being more apt not to prime than it was to prime.

Analysis of Operation and Comparison of Standard and Proposed Designs

The proposed design of siphon spillway operated with much greater consistency than the standard design. In obtaining characteristics curves, it was necessary to average a greater number of points to obtain the plotted ones in the case of the standard design; with the proposed design the spread of points to be averaged was much narrower.

The manner in which the crest heads increased with decreasing deflector

FIGURE 11

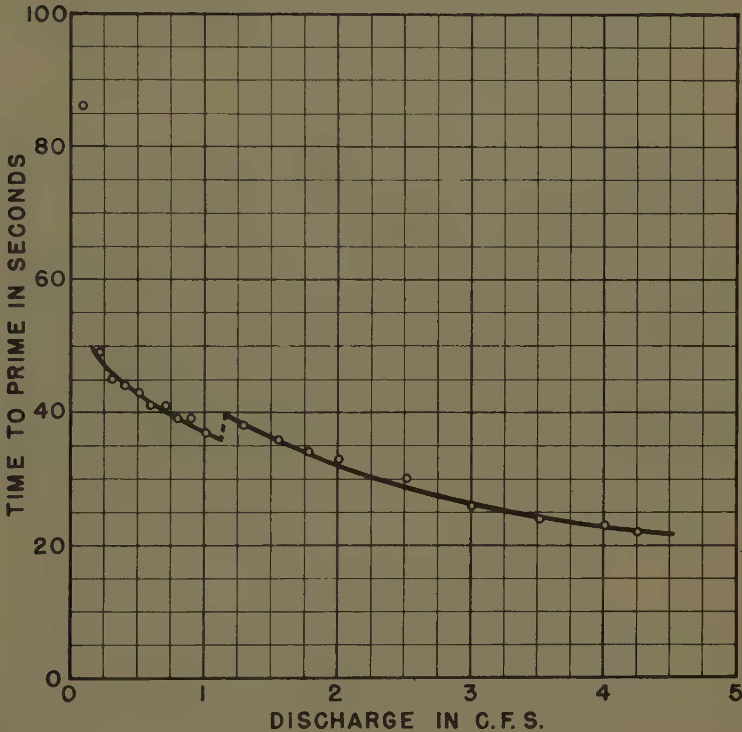


SIPHON SPILLWAY STUDIES

PROPOSED SIPHON SPILLWAY DESIGN

CREST HEAD VERSUS DISCHARGE

FIGURE 12



SIPHON SPILLWAY STUDIES

PROPOSED SIPHON SPILLWAY DESIGN

DEFLECTOR ANGLE 45°

TIME TO PRIME VERSUS DISCHARGE

INITIAL CYCLE OF OPERATION

586

angles, raises a question as to whether the heads for 50 or 55 degrees might be lower yet. There must be a practical limit, of course, and we were unable to increase the angle accurately without rebuilding the section containing the deflectors.

The principal advantage of the proposed design over the standard design is the significant reduction in the time to prime. The lag between the time water flows over the crest and air pumping begins having been nearly eliminated, efforts to fit the new siphon with a partialization device may meet with more success than was had with the standard siphon.

The new design also is capable of priming at a lower discharge than required for priming the standard design, and it is capable therefore of partializing earlier in the discharge range, even with a siphon breaker pipe.

Future Investigations

The studies conducted to date are by no means complete. Attention will be concentrated on providing the proposed siphon design with a partialization device which will proportion satisfactorily the intake of air in the crown.

Many different avenues for investigation have occurred to us which should be explored to make the studies complete. The slope of the downstream leg of the proposed design should be varied, as well as the position along the slope of the deflector. The shape of the bucket and elevation of the downstream lip undoubtedly influence the total operating head; a separate study could be made of these features.

It is believed that the new design, even with conventional siphon breaker pipe, is an improvement over that formerly used. Efforts will be made to build a prototype structure with test facilities for the purpose of developing characteristics at the larger scale.

Journal of the
HYDRAULICS DIVISION
Proceedings of the American Society of Civil Engineers

SYNTHETIC FLOOD FREQUENCY

Franklin F. Snyder,[1] M. ASCE
(Proc. Paper 1808)

SYNOPSIS

A procedure is developed for computing the flood discharge probability associated with a given rainfall-duration-frequency pattern on natural drainage basins, non-channelized overland flow areas and areas with storm sewer drainage utilizing basin runoff-producing characteristics of area, length, slope, friction and shape. The approach is patterned after the so-called rational method and utilizes the time of concentration concept with a unit hydrograph interpretation, but recognizes and evaluates separately the effect of storage existing in all types of channels or conduits and an average rainfall-runoff relation. The variable factors in this case have been evaluated for application in the vicinity of Washington, D. C.

Time of Concentration

The time of concentration, T_c, as used herein, is based primarily on hydrograph analyses(1) and can be defined as the time from cessation of effective rainfall to the inflection point of the recession side of the resulting runoff hydrograph. It is essentially the same as the time of concentration defined as the time for runoff from the remote portion of the drainage basin to reach the point of interest. As such it constitutes the time base of an area-shape diagram as visualized in some unit hydrograph analyses.(2)(1) (See Fig. 1)

Table I gives the basic data for some of the drainage areas utilized in evaluating the coefficients and exponents of the basic equations,

Note: Discussion open until March 1, 1959. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1808 is part of the copyrighted Journal of the Hydraulics Division, Proceedings of the American Society of Civil Engineers, Vol. 84, No. HY 5, October, 1958.

1. Hydraulic Engr., Office of the Chf. of Engrs., Washington, D. C.

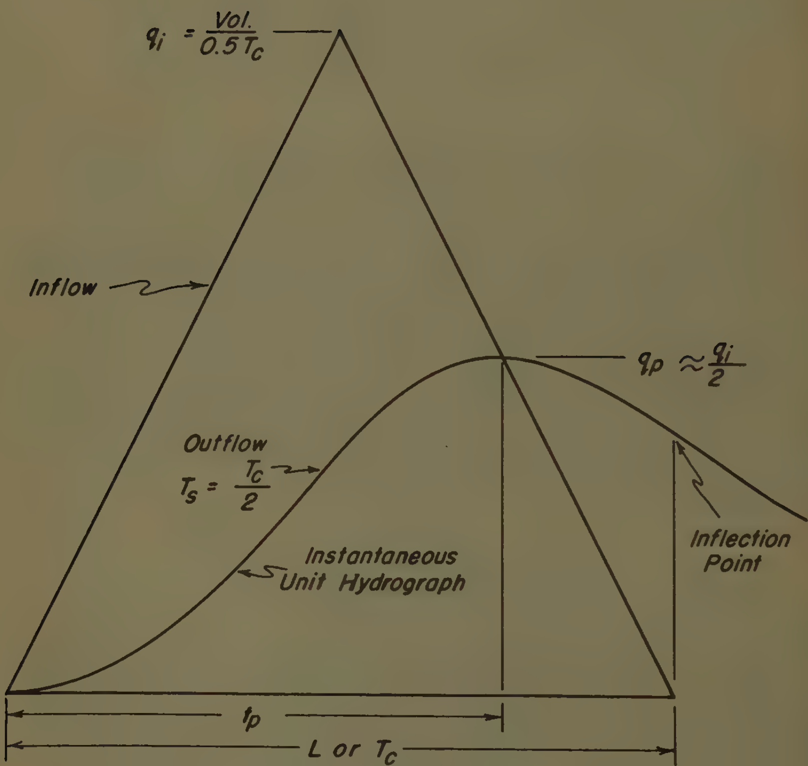


Figure 1, AREA-SHAPE DIAGRAM

TABLE I PERTINENT DATA FOR VARIOUS DRAINAGE AREAS

Drainage Basin (1)	Area (2)	Length (3)	Slope (4)	Friction (5)	T_s (6)	Obs. T_c (7)	Obs. C_t (8)	q_p T_c (9)
NATURAL AREAS	Sq.Miles	Miles	%	n	hours	hours	hrs/ mile	cfs-hrs /sq.mi
N.E. Br. Anacostia R. nr. Riverdale, Md.	72.8	15	0.54	0.07	9	14	2.8	442
N.W. Br. Anacostia R. nr. Colesville, Md	21.3	7	0.36	0.05	2.5	6	2.1	786
Wills Cr. at Cumberland, Md.	247	35	0.81	0.05	14	10	1.7	410
Little Falls Br. nr. Bethesda, Md.	4.11	2.7	1.0	0.035	0.5	1.5	1.5	640
Goose Cr. nr. Leesburg, Va.	338	33	0.22	0.06	9	22	2.3	686
Occoquan Cr. nr. Occoquan, Va.	546	40	0.11	0.06	9	25	1.9	600
Rappahannock R. nr. Fredericksburg, Va.	1600	78	0.20	0.05	16	24	1.6	566
AIRFIELD DRAINAGE (8)	Acres	feet	%	n	minutes	minutes	min./ft	cfs-min/ acre
Freeman A (22% Pavement), Ind.	9.57	625	0.56	0.40	75	38	0.29	24
Freeman D (All Turf), Ind.	9.54	1200	0.50	0.45	93	68	0.32	28
St. Anne #2 (All Turf), Ind.	38.4	1500	0.40	0.40	105	66	0.27	24
Godman #1 (22% Paved), Ky.	13.1	900	1.3	0.45	90	45	0.33	28
Freeman B (All Paved) Apron, Ind.	8.23	550	0.50	0.014	5.0	6.5	0.39	49
Freeman B Plus Taxiway	0.71	650	0.27	0.014	-	-	-	-
Freeman B Total	8.94	1200	-	-	7.0	9.0	0.31	44
Lockbourne #3 (All Paved)140' strip, Ohio	0.57	140	0.88	0.016	1.5	2.5	0.37	55
Lockbourne #3 Plus 30' strip	0.12	165	0.40	0.020	-	-	-	-
Lockbourne #3 Total	0.69	305	-	-	2.3	4.0	0.30	56
AREAS WITH STORM SEWERS (9)	Sq.mi.	miles	%	n	hours	hours	hrs/mi	cfs-hrs/ sq.mi.
17th St. at N.W. Parkway, Louisville, Ky.	0.22	0.91	0.32	0.019	0.18	0.28±0.1	0.58	705
N.W. Trunk at Shawnee Pk, Louisville, Ky.	1.90	2.91	0.10	0.013	0.35	0.50±0.1	0.45	588
Western Outfall, Broadway nr. W. P'kway	2.77	4.22	0.050	0.017	0.60	0.73±0.1	0.36	415
Southern Outfall at State Fair Grounds	6.43	6.44	0.059	0.015	0.60	1.0 ±0.1	0.42	605
S.W. Outfall, Cane Rd. nr. Camp Gd. Rd.	7.52	6.48	0.085	0.013	0.55	0.73±0.1	0.39	622
Beargrass Cr., Above Main St. and below /1 Payne St. & Ind. Ave., Louisville, Ky.	6.30	4.0	0.085	0.025	0.70	0.90	0.41	484

/1 Storm sewers with principal channel of concrete

$$T_c = C_t \, L'^{0.6} \tag{1}$$

$$L' = \frac{Ln}{0.1\sqrt{s}} \text{ or } \frac{10\,Ln}{\sqrt{s}} \tag{2}$$

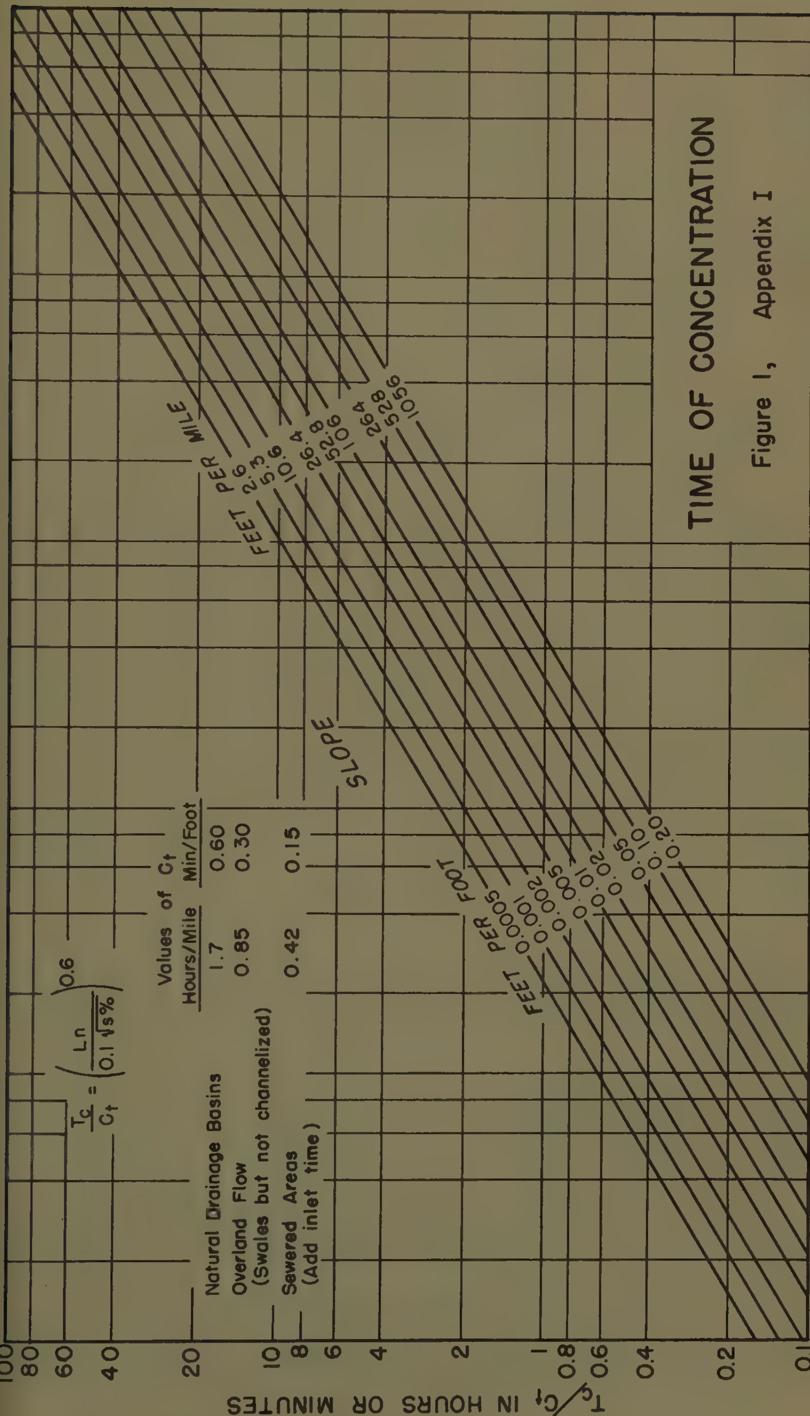
in which T_c is the time of concentration, C_t is a coefficient dependent on the type of drainage system, L' is the length of an equivalent channel or conduit having the same time of concentration but with a standard slope of 1% and a standard friction factor of 0.1, L is the length of the principal channel, s is the weighted slope of the principal channel, in %, determined as the mean height of the channel profile above the point of interest divided by one-half the length, and n is the friction factor for the channel or conduit.[2] Fig. 1 of Appendix I provides a graphical solution of Eq. 1. Column 8 of Table 1 presents observed values of C_t for the drainage areas listed, which can be compared with the values of C_t as given in Appendix 1 for use in the vicinity of Washington, D. C.

The general form of Eqs. 1 and 2 were assumed, and varied and successive plottings of the data in Table I were utilized to evaluate the exponents and coefficients. Assuming slope to the 0.5 power, values of L' were plotted against values of T_c on logarithmetic paper. The best exponent was slightly greater than 0.6 for natural areas, slightly less than 0.6 for the airfields and about 0.5 for the sewered areas. A value of 0.6 is consistent with that developed by the author[2] from a study of a large number of unit hydrographs and was adapted for use for all three types of drainage. Values of length to the 0.6 power divided by T_c were also plotted against values of slope. The best exponent was about 0.4 for natural areas and not definable for the airfields and sewers, partly because of the limited range of slopes available. By including the slope factor in the expression for L', an actual value of 0.3 results for the slope-effect exponent. Use of this value for slope effect did not weaken the overall correlation for the airfield and sewered areas. Average, equivalent and weighted values of slope were tested, and it was found that the weighted form, as adopted, gave the best correlation. Consideration was also given to the use of length to the 0.6 power rather than equivalent length, L', but the latter gave slightly better results. A value of 0.10 was used as the standard friction factor, n, in converting actual lengths to equivalent lengths, L'. An overall value of about 2.0 for C_t for natural basins was evident but a value of 1.7 was selected to be slightly conservative for use in the vicinity of Washington, D. C. on the basis of values for the natural basins in Table I.

The value of C_t of 1.7 hours per mile for natural drainage basins in the vicinity of Washington, D. C., is four times larger than that for completely sewered areas. The expression for L' takes into account all the factors of the Manning equation except the hydraulic radius which is indirectly recognized in the expression for T_c through the exponent, 0.6. The question thus arises as to why the value of C_t for natural basins should be so much larger than that for sewered areas.

It is believed that this difference, as well as much of the differences between values of T_c for various natural basins, results from differences in the amount of subsurface storm-flow that exists in representative hydrographs

2. The letter symbols in this paper are defined where they first appear in the text and are given in Appendix II for ready reference.



TIME OF CONCENTRATION

Figure I, Appendix I

of direct runoff for the areas. Direct runoff can be defined as that part of the total runoff appearing systematically in the drainage channels in direct response to rainfall or melting snow in contrast to ground-water or base flow which enters the channels from the main zone of saturation. Direct runoff thus includes all or most of the true surface runoff as well as an appreciable portion of the subsurface storm-flow which is characteristic of the area. Subsurface storm-flow can be defined as that portion of the storm flow which penetrates the surface of the ground temporarily but soon reappears or drains away from the area through the upper soil layers more rapidly than true ground-water seepage.

As such, subsurface storm-flow is subject to greater storage effect and time delay than is true surface runoff. The direct runoff hydrographs for a drainage basin thus reflect some subsurface as well as surface storage characteristics.

Except for areas of 100 per cent imperviousness, storm runoff hydrographs from urban areas with complete storm drainage systems contain some subsurface storm-flow from yards, banks, stone driveways, etc. For such areas, however, the possible range in the relative amount of subsurface storm-flow is much smaller than that for natural drainage basins. After a value of C_t for natural basins has been selected for a particular region, it is considered that the percentage of open drainage channels which have been eliminated and the percentage of the basin which has been storm sewered are the principal factors to use for interpolating between the selected value of T_c for the natural basin and the value of 0.42 for the complete storm drainage condition. Data are not available to define the relationship exactly, but those which are available indicate that reasonable results are obtained by averaging the two percentage factors and interpolating on a straight line between C_t's (1.7 in the case of the vicinity of Washington, D. C.) and 0.42. Thus, if 80 per cent of a basin has storm sewers, but only 40 per cent of the natural drainage channels have been eliminated, a value of C_t of 0.9 is appropriate, that is, 1.7 minus $\frac{0.80 + 0.40}{2}(1.7 - 0.42)$.

Rainfall-Duration-Frequency

Fig. 2, Appendix I, shows the rainfall-duration-frequency curves of point rainfall recommended for use in the vicinity of Washington, D. C. These curves are based on data published by the U. S. Weather Bureau in Technical Paper No. 25.(3)

The rainfall intensity-duration-frequency curves of T. P. No. 25 were adjusted, as recommended therein, to give partial-duration values and the rates were converted to amounts.

The curves presented in T. P. No. 25 for Washington, D. C. were derived from the annual series (maximum value for each year) for the periods 1896-1897 and 1899-1953, and are spaced according to the Gumbel procedure.

Also shown on Fig. 2, Appendix I, are curves giving factors to apply to values of point rainfall to obtain average depth of rainfall over drainage basins of appreciable size.

A dense network of recording rain gages with a reasonably long period of record is required to determine the intensity-duration-frequency relationship of areal rainfall to that of a centrally located point observation. The Weather

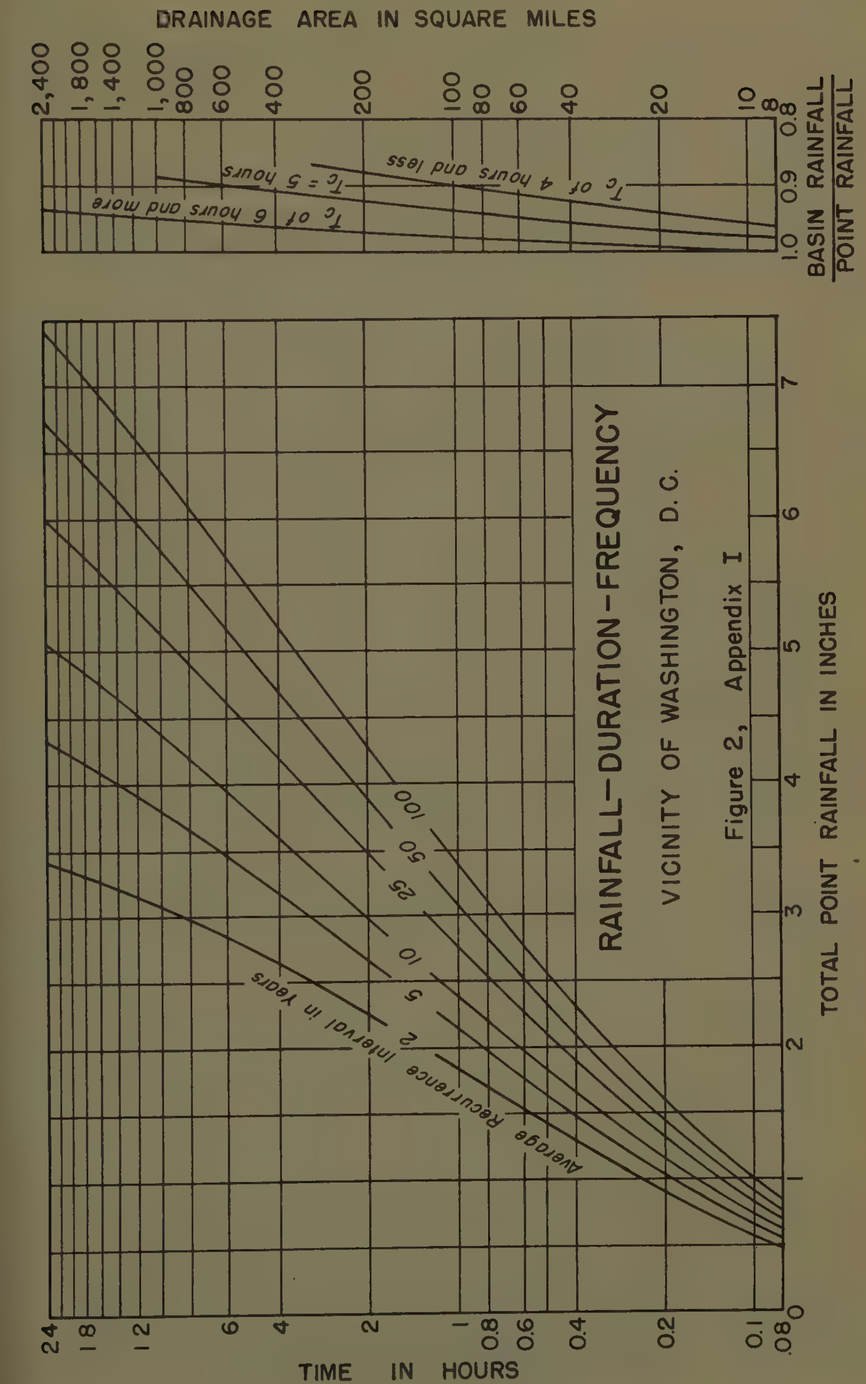


Figure 2, Appendix I

Bureau has published the results of an investigation of this problem in Technical Paper No. 29.[4] Seven networks having the most suitable density of gages and length of record were selected from all available networks of this type. Even on this selective basis, the length of record available averaged only about 12 years. The results were quite scattered but average relationships were drawn for durations of 1/2; 1- and 24-hours with intermediate values interpolated.

A study by the State of Illinois[5] of one of their dense networks of rain gages indicated a considerably smaller reduction than the Weather Bureau for areas up to 100 square miles and a duration of 6 hours. Part of the difference may be due to differences in the methods of analyses.

Because of the lack of comprehensive information on this matter, it is considered that the reduction to point rainfall values for size of drainage area should be estimated conservatively. On this basis, it is proposed that use be made of average relationships between point rainfall at storm centers and the maximum average depths over increasingly larger areas which can be determined for a given region from storm study data such as that published by the Corps of Engineers.[6] In general, two relationships between maximum point rainfall and average depth over surrounding areas are apparent, one for intense thunderstorm rainfall of short duration and limited extent (summer type) and one for widespread general storms (winter type). The summer type rainfall can be considered to control areal variation for durations less than four hours with the winter type rainfall controlling for durations greater than six hours.

If it is assumed that all storms are widespread enough to cover areas equal to or larger than an area of interest, a rational adjustment to the relationships based on storm center rainfall can be evolved. On the basis of a random distribution of storm centers over a circular shaped drainage basin, it can be reasoned that the average reduction factor to a fixed-point record typical of a particular drainage basin should be about one-third that based on storm center rainfall. The following results are obtained:

Size of Area, Square Miles	Reduction Factor to Point Rainfall, %	
	Summer Type	Winter Type
1	100	100
10	96	100
100	90	98
1,000	83	95
10,000	--	92

It is recognized that small summer thunderstorms often cover only a few square miles and thus affect the intensity-duration-frequency relationship at point locations with little or no effect on moderate sized drainage basins. Disregard of this effect, which would tend to decrease the above reduction factors, is considered to be conservative, but appropriate in view of the present state of knowledge of the matter.

In a region where topographic or other factors produce a variation in annual rainfall, recognition must be given to these differences in selection

or adjustment of the point rainfall record to be used. Average annual rainfall serves as a satisfactory basis for adjustment within a region of similar rainfall characteristics.

Rainfall-Runoff-Relationship

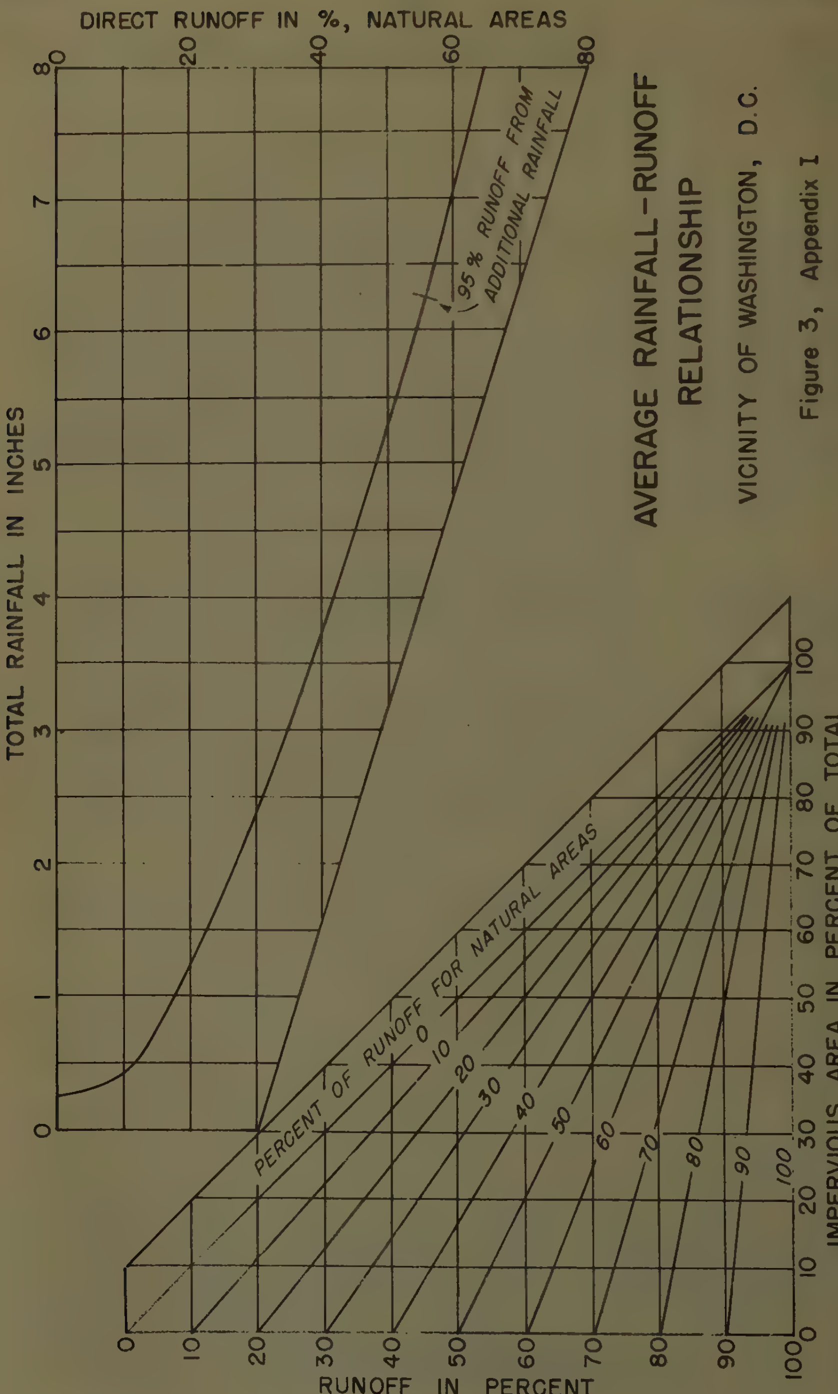
The average rainfall-runoff relationship shown on Fig. 3 of Appendix I for the vicinity of Washington, D. C., is based on rainfall-runoff data for natural drainage basins in that area. The long time average annual rainfall at Washington is about 41.5 inches. According to records of the U. S. Geological Survey the average annual runoff of gaged drainage basins in the vicinity of Washington varies from about 12 to 18 inches per year. In the rare cases where measurements of runoff are not available for drainage basins in the vicinity of or typical of the point of interest, estimates of average annual runoff can be obtained from climatological maps or estimated from climatological factors. On the basis of a simple relationship, developed by the author, giving the annual loss (precipitation minus runoff) for humid areas as

$$\begin{aligned} \text{Loss} &= 0.6\ (\text{Average temperature} - 11) \\ \text{Loss} &= 0.6\ (56.8 - 11) = 27.5 \text{ inches,} \end{aligned} \tag{3}$$

the annual runoff in the vicinity of Washington, D. C. would be 14 inches. Eq. 3 is based on records of about 30 drainage basins scattered over the humid section of the United States with an average error of 1.8 inches of runoff.

The desired relationship between rainfall and direct runoff is one for average conditions on the assumption that storms of any given frequency will occur on various conditions of runoff potential, and over a period of years the average result would be that for average runoff conditions. One alternative would be to develop rainfall-runoff relations in terms of rainfall duration, amount, season of the year and antecedent conditions. Introduction of the frequency factor would require analysis of a large amount of data and a number of simplifying assumptions. Application of the results of such an analysis would require a reverse application of the assumptions and selection of specific values of the variables for use in synthesizing a flood frequency relationship. A practical alternative for natural basins would be to compute the actual runoff for the two or three probable maximum flood producing storms each year for a number of years under the actual associated soil conditions and develop an annual flood series. It is believed that the difficulties associated with these alternatives would leave the resultant flood frequency relation open to more uncertainties than the procedure proposed herein.

The rainfall and runoff data were plotted on an average monthly basis and a mean curve drawn to give slightly higher runoff than the average, starting with an initial loss of 0.25 inch. Although it would at first appear that the relationship for isolated storms should give considerably more direct runoff than average monthly data, it must be remembered that in some drainage basins as much as half of the total annual runoff is ground-water discharge, whereas during isolated flood events the percentage of the peak discharge contributed by ground water is much smaller. The upper end of the curve was controlled by assuming all losses (2.75") satisfied (except for a continuous contribution of 5 % to ground water) with a total rainfall of 6.25 inches. These values were based on experience gained from detailed rainfall-runoff



AVERAGE RAINFALL-RUNOFF RELATIONSHIP

VICINITY OF WASHINGTON, D.C.

Figure 3, Appendix I

investigations of similar areas. The average rainfall-runoff curve was then converted to a per cent runoff curve for more convenient use.

The curves of Fig. 3, Appendix I, for adjusting the percentage of runoff on natural areas to obtain the percentage of runoff on a developed area, taking into account the portion of the area that is impervious were computed with the equation:

$$\text{Adj. RO \%} = \text{\% Impervious Area} + \frac{(100 - \text{\%Imp. Area})\ \text{\%Nat. R.O.}}{100} \qquad (4)$$

The equation assumes 100 per cent runoff from the impervious area. Wetting of the surface and ponding of water in small depressions and behind various surface irregularities can easily account for from 0.05 to 0.10 inch of water even on a fairly smooth airfield pavement. However, assuming 100 per cent runoff provides a small factor of safety and is the only such unevaluated one in the procedure described herein. Furthermore, in many instances the depression storage may be filled by antecedent rainfall.

Effect of Storage and Rainfall Duration

Fig. 1 and the following discussion illustrate the conditions and assumptions under which the so-called "rational method", when used with a proper runoff coefficient, results in reliable estimates of discharge, even though the storage effects of the particular conduits involved are not evaluated separately.

The isosceles triangle of Fig. 1 with a time base of T_c is assumed as the area-shape diagram of a particular area. The diagram represents the distribution of the area with respect to distance in time (or length) away from the point of interest. If 1 inch of water were deposited instantaneously or within a very short time over the whole area and the water moved at a constant velocity of L/T_c down the drainage paths to the point of interest, the hydrograph observed would be the equivalent of the area-shape (discharge) diagram (1 acre-inch = 1 cfs-hour) under the "rational method" concept of time of concentration. For many typical drainage basins an isosceles triangle can be accepted as a reasonably close approximation of the actual area-shape diagram. Assumption of a rectangular area-shape would change the value of q_p in Eq. 8 by only 15%.

The use of an instantaneous rainfall simplifies the following analysis of the assumptions inherent in the "rational method". However, if a finite rainfall duration t were used for the hypothetical drainage area of Fig. 1, the area-shape diagram would become essentailly a trapezoid with top width t, and bottom width $t + T_c$. So long as t is made about one-seventh or smaller part of T_c the results of this analysis would be affected very little.

Water cannot flow without first building up depth, and with depth comes a storage effect (also variable velocity) on discharge. Accordingly, observed hydrographs of discharge for unit rainfalls are not area-shape (discharge) diagrams but rather are area-shape (discharge) diagrams modified by the storage and velocity or wave movement characteristics of the collecting media (2).

The effect of drainage channel storage on discharge from headwater areas has been found to be closely approximated by reservoir type storage(1) of an

amount

$$T_s = \frac{\Delta S}{\Delta Q} \tag{5}$$

where T_s is an index of storage effect with the dimension of time. T_s can be evaluated from the recession side of discharge hydrographs after inflow has essentially ceased by the equation, $T_s = -t/\log_e C_r$) where C_r is a recession coefficient, Q_2/Q_1, for two values of discharge separated by time, t. Because direct runoff recessions can only be estimated by separation from total flow for discharge hydrographs of natural drainage basins, values of T_s so determined are subject to a number of inaccuracies as well as the possibility of variation with season of the year and size of flood.

Referring again to Fig. 1 and assuming a unit area of 1 acre with 1 inch of rain deposited thereon and time in hours,

$$q_i = \frac{Vol}{0.5T_c} = \frac{1 \text{ cfs-hr}}{0.5T_c} \quad \left(= \frac{645 \text{ cfs-hrs}}{0.5T_c} \text{ for 1" on 1 sq. mile}\right) \tag{6}$$

and if $T_s = T_c/2$,

$$q_p \approx q_i/2 \tag{7}$$

in which q_i is the peak of the isosceles triangular area-shape (discharge) diagram in cfs-hours per acre and q_p is the peak of the discharge hydrograph per unit of area (per acre) per unit of runoff obtained by routing the area-shape diagram through a reservoir type storage of $T_s = T_c/2$ hours.

Then from Eq. 6

$$q_p = \frac{q_i}{2} = \frac{Vol.}{T_c} = \frac{1}{T_c} \tag{8}$$

and

$$q_p T_c = 1 \quad (1.01 \text{ for 1 acre or 645 for 1 sq. mi.}) \tag{9}$$

Under the assumption of one inch of rain in a short period the outflow hydrograph with peak q_p, is an instantaneous unit hydrograph per unit of area for the basin with ordinates in cfs per acre. A unit hydrograph for a given area may be defined as the discharge hydrograph of direct runoff resulting from 1 inch of runoff from a rain occurring in a specified unit of time. The unit hydrograph procedure is now the most commonly used procedure for synthesizing discharge hydrographs.

The peak discharge, Q_p, for a drainage basin after the unit hydrograph procedure using the value of q_p from Eq. 8 is

$$Q_p = K_d A \frac{1}{T_c} I_r T_c = K_d I_r A, \tag{10}$$

where A equals the area in acres, I_r is the average rate of runoff over the time of concentration, T_c, in inches per hour or the equivalent of Ci in the rational approach, Q_p = CiA, and K_d is a reduction factor for adjusting the result, in that the rainfall and resulting runoff, I_r, is the mean rate over the time of concentration, T_c, rather than over the very short period of time assumed in developing the unit hydrograph peak, q_p.

Thus it has been shown that, with the assumption of an isosceles area-shape diagram and a basin storage factor of $T_s = T_c/2$, that the rational approach gives the same result as use of a unit hydrograph except for the factor K_d. With the effective rainfall spread over a duration of T_c hours in

the rational method, the value of K_d is 0.87 for a typical unit hydrograph shape and rainfall distribution of duration, T_c.

The value of K_d was evaluated as 0.87 by applying a typical unit hydrograph shape to a typical rainfall distribution over the time of concentration and comparing the resulting peak discharge with that which was obtained by assuming the same total effective rainfall in one period. The rainfall distribution in per cent of total for a time of concentration divided into seven equal periods was: 4, 8, 10, 20, 40, 12 and 6%. These numerical values are typical for durations or times of concentration of from one to six hours and the arrangement is critical for unit hydrograph usage. For longer durations the shape would be somewhat more concentrated.

However, area-shape diagrams for actual drainage basins vary somewhat from isosceles triangles. The variation can be measured by the ratio $L_{ca/L}$ (equals 0.5 for an isosceles triangle) where L_{ca} is the distance along the principal channel to a point opposite the center of area of the basin. Also, although $T_s = T_{c/2}$ is fairly representative of the storage effect of many drainage channels, the value of T_s does vary from basin to basin and from conduit to conduit. Accordingly, equations (8) and (9) need adjustment in cases where the actual values of T_s and $L_{ca/L}$ vary considerably from $0.5\ T_c$ and 0.5, respectively. This can be accomplished as follows:

$$q_p T_c = K_s \, 1.03 \, \underline{/1} \tag{11}$$

in which

$$K_s = \frac{2/3\ (1 + L_{ca/L})}{T_{s/T_c} + 0.5} \tag{12}$$

Eq. 12 was developed by routing additional triangular area-shape diagrams of other than isosceles character (variable values of $L_{ca/L}$) with various values of T_s/T_c and evaluating the resulting effect of the variables on q_pT_c. Although more approximate in the case of area-shapes differing from the triangular shape assumed in developing the relationship, Eq. 12 serves as a practical adjustment for other typical shapes.

For the Washington, D. C. area a value of $K_s = 0.86$ is appropriate for the natural areas. The resulting value of 0.89 for q_pT_c is also applicable to sewered areas and to paved surfaces with apparently little limitation as to location. For non-paved overland flow surfaces a value of q_pT_c of 0.50 is satisfactory. Values of q_pT_c in Table 1 would be about 570 cfs-hours per square mile and 53 cfs-min/per acre for a value of K_s of 0.86 and about 320 cfs-hours per square mile and 30 cfs-min/per acre for a value of K_s of 0.49.

The adjustment factor, K_s, of 0.86 for storage effects and basin shapes, when multiplied by 1.03 and the adjustment factor of 0.87 (K_d) for an effective rainfall duration of T_c, gives a value of 0.77 (0.44 for overland flow on turf) as the total adjustment factor, K, to be used in

$$Q_p = KAI_r \tag{13}$$

where

$$K = 1.03\ K_d\ K_s$$

Note: 1.03 (658 when area is in square miles) is the actual value of q_pT_c for an isosceles shape and $T_s/T_c = 0.5$.

APPLICATION

Appendix I outlines the step by step procedures to be followed in computing flood frequencies by the procedures described herein with the variable factors for natural drainage basins evaluated for use in the vicinity of Washington, D. C. Table 2 gives sample computations for Goose Creek near Leesburg, Va., Occoquan Creek near Occoquan, Va., Accotink Creek near Annandale, Va., and Little Falls Branch, Bethesda, Md. The observed values of flood probabilities given for comparison are based on U. S. Geological Survey data with 26 years of record for Goose Creek, 25 years for Occoquan Creek, 10 years for Accotink Creek and 12 years for Little Falls Branch. It should be noted that the synthetic frequencies constitute a partial duration series and as such the values for recurrence intervals of 10 years and less should be somewhat larger than the annual series values with which they are compared.

In cases where it is desired to compute the discharge at a large number of locations for a selected frequency of occurrence such as in the design of storm sewer systems, working curves to expedite the computations can be developed on the basis of the procedures described herein. Using Eq. 13 values of KI_r, K = 0.77 for the vicinity of Washington, D. C., for a specific frequency are plotted against indicated values of T_c with a family of curves covering the desired range of imperviousness. Since a K value of 0.77 is adequate for natural areas, overland flow on pavements, and sewered areas, one set of curves such as Fig. 4 of Appendix I is all that is needed for each frequency of interest in most cases. If conditions of overland flow on turf are involved, values of KI_r read from the zero or other appropriate imperviousness curve, reduced by 0.57 ($^{0.44}/0.77$), are applicable. If overland flow on turf is of primary interest, separate KI_r curves for this condition can be prepared.

The initial collection or inlet time at the head of each line can be computed by use of appropriate factor (C_t = 0.30) or estimated on a basis of judgment (ordinarily about 10 minutes). Because the value of C_t for overland flow is twice that for sewered areas, the computation of inlet time is kept separate from the computation of T_c for the sewered portion of the collection system, and the inlet time is added each time to the computed value of T_c for downstream sections of the line. After a point is reached where the time of concentration is greater than about 2 hours, retention of the inlet time as an additive factor can be stopped as a practical measure.

In cases where storage is available along a drainage system such as at an inlet for airfield drainage or at a pumping plant, the peak discharge as obtained by the procedures of this paper can be adjusted for the effect of the storage available. This is accomplished by adding to the computed time of concentration, T_c, for the location, 1.8 times the value of T_s determined by successive trials as the average over the range of outflow and available storage. This is accomplished by assuming that the peak discharge for the location will be reduced by the ratio of the available storage to the volume of the runoff. The available storage is then divided by the trial regulated discharge so determined to get a trial value of T_s. The regulated peak discharge is then computed for the location by the regular procedure using an adjusted time of concentration equal to the original $T_c + 1.8T_s$. If the resultant peak discharge is not sufficiently close to the assumed trial value,

TABLE 2A COMPUTATION OF SYNTHETIC FLOOD FREQUENCIES

OCCOQUAN CREEK NEAR OCCOQUAN, VA.

Natural basin but with several reservoirs which reduce flood peaks slightly.

Dr. A. = 546 sq. miles; L = 40 miles; s = 5.6'/miles = 0.11%; n = 0.06
Annual basin rainfall is $\lessapprox$ Washington rainfall; Adj. for size = 0.96

$T_c = 1.7 \left(\frac{10 \times 40 \times .06}{\sqrt{0.11}}\right)^{0.6} = 22.4$ hours

Adj. for effect of reservoirs + 1.6

Adj. T_c = 24 hours

$Q_p = 500\ AI_r = 273{,}000\ I_r$

Freq. (Recur. Int., Yrs.)	2	5	10	25	50	100
Point Rainfall, inches	3.42	4.33	5.05	5.97	6.72	7.40
Basin Rainfall, inches	3.28	4.16	4.85	5.73	6.45	7.10
Percent RO, Natural	37	43	47.5	53	57	60
Runoff, inches	1.21	1.79	2.31	3.04	3.67	4.26
I_r, inches per hr.	.050	.074	.096	.127	.153	.178
Q_p, cfs	13,800	20,300	26,200	34,600	41,800	48,500
Obs. 25 Yr. Record /1	13,500	20,000	24,400	30,000	34,000	(38,000)
Obs. 25 Yr. Record /2	12,700	20,000	25,700	34,000	41,000	(48,500)

ACCOTINK CREEK NEAR ANNANDALE, VA.

Dr. A. = 23.6 sq. miles; L = 11.0; s = 0.29; n = 0.05

$T_c = 1.7 \left(\frac{10 \times 11 \times .05}{\sqrt{0.29}}\right)^{0.6} = 1.7 \times 4.03 = 6.8$ hours

Assume 5% Average Imperviousness, 1947-1956

Areal Adj. less than 1% - Ignore

$Q_p = 500\ AI_r = 11{,}800\ I_r$

Freq. (Recur. Int., Yrs.)	2	5	10	25	50	100
Point Rainfall, inches	2.91	3.53	4.06	4.74	5.31	5.87
Percent RO, Natural	34	38.5	42.3	46.5	50.2	53.8
Percent RO, Adjusted	37.3	41.6	45.2	49.2	52.7	56.0
Runoff, inches	1.08	1.47	1.83	2.33	2.80	3.29
I_r, inches per hr.	0.160	.216	.270	.343	.412	.484
Q_p, cfs	1880	2550	3180	4050	4860	5700
Obs. 10 Year Record /1	1550	2530	3200	4000	(4600)	(5220)

/1 U.S. Geological Survey records, after Gumbel
/2 Best fit, with recurrence interval = (N+1)/M

TABLE 2B COMPUTATION OF SYNTHETIC FLOOD FREQUENCIES

LITTLE FALLS BRANCH NEAR BETHESDA, MD.

Principal channels mostly open but with numerous storm sewers.

Dr. A. = 4.11 sq. mi.; L = 2.75 mi.; s = 53'/mile = 1.0%; n = 0.045

As of 1950 about 22% of area was impervious.

10% natural channels eliminated; 30% of area storm sewered.

$C_t = 1.7 - 0.2\ (1.7 - 0.42) = 1.44$

$T_c = 1.44 \left(\frac{10 \times 2.75 \times .045}{\sqrt{1.0}}\right)^{0.6} = 1.65$ hours

$Q_p = 500\ AI_r = 2055\ I_r$

Freq. (Recur. Int., Yrs.)	2	5	10	25	50	100
Rainfall, inches	2.15	2.51	2.82	3.27	3.65	4.02
Percent RO, Natural	28	31	33.5	37	39.5	42
Percent RO, Adjusted	44	46	47.5	51	53	55
Runoff, inches	0.95	1.15	1.34	1.67	1.94	2.21
I_r, inches per hr.	0.574	0.70	0.813	1.01	1.17	1.34
Q_p, cfs	1180	1440	1670	2080	2410	2760
Obs. 12 Yr. Record /1	1040	1440	1710	2060	(2300)	(2570)

GOOSE CREEK NEAR LEESBURG, VA.

Dr. A. = 338 sq. miles; L = 33 miles; s = 11.4'/mile = 0.22%; n = 0.06

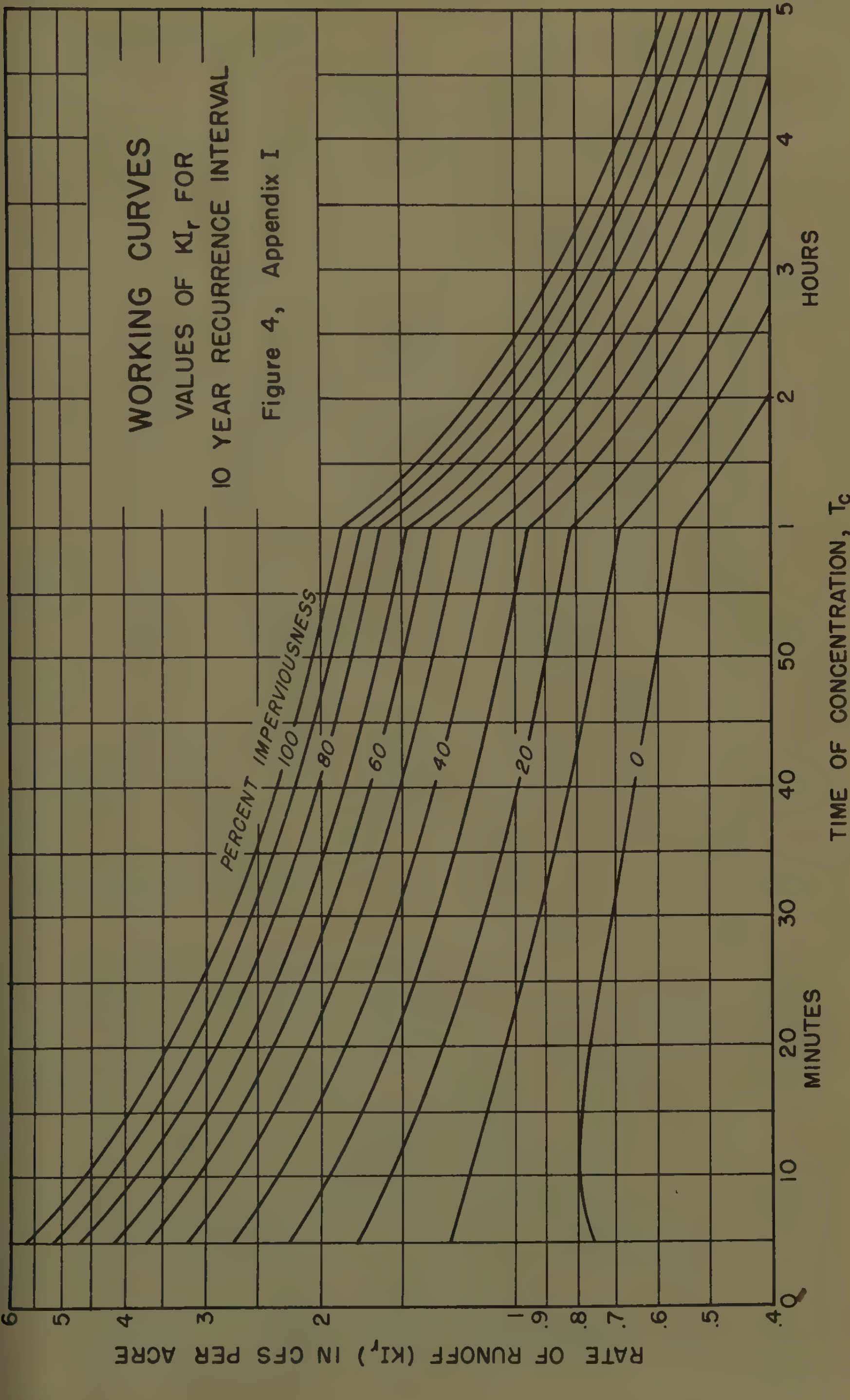
$T_c = 1.7 \left(\frac{10 \times 33 \times .06}{\sqrt{0.22}}\right)^{0.6} = 16.2$ hours

Annual Basin rainfall is ≂ Washington rainfall; Adj. for size = 0.96

$Q_p = 500\ AI_r = 169{,}000\ I_r$

Freq. (Recur. Int., Yrs.)	2	5	10	25	50	100
Point Rainfall, inches	3.27	4.09	4.75	5.60	6.30	6.95
Basin Rainfall, inches	3.12	3.92	4.56	5.38	6.05	6.67
Percent RO, Natural	35.5	41	45.5	51	55	58.5
Runoff, inches	1.11	1.68	2.07	2.75	3.33	3.90
I_r, inches per hr.	0.068	.103	.128	.170	.206	.241
Q_p, cfs	11,500	17,500	21,600	28,700	34,800	40,700
Obs. 26 Yr. Record /1	10,000	20,400	27,300	36,000	42,500	(49,000)

/1 U.S. Geological Survey records, after Gumbel



WORKING CURVES

VALUES OF KI_r FOR

10 YEAR RECURRENCE INTERVAL

Figure 4, Appendix I

the computation is repeated using the computed regulated peak discharge to get a new trial value of T_s. A similar adjustment was made in Table 2 for Occoquan Creek to adjust for the effect of existing reservoirs.

Values for the coefficient, C_t, and adjustment factor, K_s, presented herein for overland flow areas and areas with storm sewer drainage, are considered to have fairly wide application. The range of slope, s, for the drainage areas listed in Table I is perhaps not great enough to fully support such an assumption. However, it is believed that some extrapolation with respect to this characteristic is permissible, particularly in view of the fact that slope is effective to the 0.3 power only.

The time and storage factors for the overland flow areas of Table I are based on hydrographs of runoff which were adjusted to eliminate the effect of storage due to ponding, channel flow to inlets, or flow in collecting pipes.(8) Adjustment for appreciable channelization or ponding should be made when synthesizing discharge values with the procedures of this paper. The time and storage factors were also found to vary with the rate of rainfall, especially on the paved areas. The values of these factors reported in Table I are averages for storms with rates of runoff varying from 0.5 to 3.0 inches per hour for the turfed areas and were selected for supply rates of about 6 inches per hour for the smaller paved areas.

The time and storage factors for the storm sewered drainage basins in the Louisville, Ky., area as listed in Table I, are based on unit hydrographs developed from observed hydrographs of runoff for isolated summer storms of short duration.(9) The full capacity of the lower sections of each of the systems was approximated in one or another of the storms analyzed, except for the Southwestern Outfall. None of the storms caused surcharge of the main trunk sewers. Main trunk shapes included were arch, egg-shaped, horseshoe, semi-elliptical and circular, but other factors obscured the relationship, if any, existing between main trunk shape and storage effect. The times of concentration, T_c, as used herein and obtained from hydrograph analyses, were smaller than those developed by the rational approach for the sewer systems, especially for the larger systems.

The value of 0.87 for K_d is based on an average storm distribution and typical unit-hydrograph shape. Rather extreme variations from the assumed typical conditions would be necessary to change this factor by as much as 10%.

Subject to the comments in the preceding paragraphs of this section and to subsequent comments with respect to rainfall-runoff relationships, it is believed that the procedures and coefficients presented herein for storm sewer systems and overland flow are satisfactory for general application and will give reasonable results in most locations.

A number of adjustments and selection or verification of coefficients are necessary in application of these procedures to natural drainage basins with unusual characteristics or ones generally different than those of the Appalachian Highlands. Suggestions for developing these adjustments are presented in the following paragraphs.

The values of C_t and K_s to be used can be determined by using Eqs. 1 and 11, if a tabulation of unit hydrograph data such as that published by the Corps of Engineers(10) is available for streams in the vicinity. If such data are not available, they can be developed from streamflow records or a time of concentration estimated by conventional procedures and a storage factor, K_s, selected from general knowledge of runoff characteristics of the region.

As previously stated the value of K_d of 0.87 would need little or no adjustment. In any event values for length, slope and friction can be measured or estimated with a satisfactory degree of accuracy for basins with streamflow records as well as for the area for which synthetic flood frequencies are desired.

On some drainage basins there is fairly conclusive evidence that the unit hydrograph characteristics vary with the magnitude of the particular flood. Where such variation can be defined, a more accurate flood frequency would be obtained by varying the values of C_t and K_s on the basis of the recurrence intervals desired.

The rainfall intensity-duration-frequency data of T.P. No. 25(3) provide a basic source of rainfall data for general use. In a number of instances, the author has obtained satisfactory results by using data from T.P. No. 25 for the station nearest or most appropriate for application to a particular drainage basin. For durations less than 4 hours little adjustment is necessary while for durations greater than 6 hours the station values can be adjusted by the ratio of the average annual rainfall for the basin to the average annual rainfall for the station. There is some indication that this results in a slight over-adjustment.

The rainfall-runoff relationship for natural areas of Fig. 3, Appendix I, has been used with fairly good results on drainage basins of the Appalachian Highlands without adjustment. It is also considered that the relationship would provide a satisfactory basis for a storm sewer design in many urban areas. However, the rainfall-runoff relationship is the factor that would require the most effort in developing adjustments for use on natural drainage basins in areas significantly different than those in the vicinity of Washington, D. C.

One basis for adjustment of the relationship is to compare the average annual runoff-rainfall ratio for the area of interest with that for the vicinity of Washington, D. C., through the medium of the average monthly rainfall for the respective areas. The relationship of Fig. 3, Appendix I, was drawn slightly to the right of the average annual runoff-rainfall ratio of 33.7 % (14.0/41.5) plotted against an average monthly rainfall of 3.46" (41.5/12). With a selected initial loss for the area of interest (0.25" for vicinity of Washington, D. C.) as one point, a relationship can be drawn on Fig. 3, Appendix I, relative to the average monthly data for the area of interest.

Another procedure for developing a rainfall-runoff relationship for use in a specific area would be to utilize available flood frequency data for streams typical of the region. The computations as illustrated in Table 2 could be made in reverse order starting with the observed flood frequencies to obtain the natural runoff percentages for plotting against the appropriate rainfall values.

ACKNOWLEDGMENT

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REFERENCES

1. "Storage and the Unit Hydrograph" by C. O. Clark, Trans. ASCE, Vol. 110, 1945, pp. 1419-1446.

2. "Synthetic Unit-Graphs" by Franklin F. Snyder, Trans. Am. Geophys. Union, 1938, Part 1, pp. 447-454.

3. "Rainfall Intensity-Duration-Frequency Curves" for Selected Stations in the United States, Alaska, Hawaiian Islands and Puerto Rico, U. S. Weather Bureau, Technical Paper No. 25, Washington, D. C., Dec. 1955.

4. Rainfall Intensity-Frequency Regime, Part I, the Ohio Valley, U. S. Weather Bureau, Technical Paper No. 29, Washington, D. C., June 1957.

5. "Rainfall Relations on Small Areas in Illinois", by F. A. Huff and J. C. Neill, Bulletin No. 44, Illinois State Water Survey Division, Urbana, Illinois, 1957, pp. 55-56.

6. "Storm Rainfall in the United States" Corps of Engineers, U.S.A., 1945, with supplements.

7. "Studies of Relations of Rainfall and Runoff in the United States," by W. G. Hoyt and others, W. S. P. 772, 1936, pp. 123-133.

8. "Report on Drainage Verification at Military Establishments" Louisville District, Corps of Engineers, Louisville, Ky., 1947.

9. "Supplemental Report, Definite Project for Local Flood Protection, Louisville, Ky., Sewer Runoff Investigation," Louisville District, Corps of Engineers, Louisville, Ky., 1949.

10. Unit Hydrograph Compilations, Project CW 153, Washington District, Corps of Engineers, Washington, D. C., 3 Volumes 1949, 1 Volume 1954.

APPENDIX I - PROCEDURE

1. Determine T_c

$$T_c = C_t L'^{\,0.6} \qquad L' = \frac{10\,Ln}{\sqrt{s}}$$

Solve equation or use Fig. 1

L = length of principal drainage channel
s = weighted slope of principal drainage channel.
= mean height of channel profile divided by L/2
n = friction factor

Average Natural Channels, overall	0.05
Paved Channels	0.014 to 0.040
Overland Flow and Improved Channels	
Smooth paving	0.016
Bare packed soil	0.10
Poor grass cover	0.30
Average grass cover	0.40
Dense grass cover	0.50
Storm Sewers	0.012 to 0.025

(Add time of overland flow (inlet time) for small areas where it is a significant factor)

Values of C_t	Hours/mile	Min./foot
Natural Drainage Basins	1.7	0.60
Overland Flow	0.85	0.30
Sewered Areas (Add inlet time)	0.42	0.15

Partially Sewered Area C_t Adj. $= 1.7 - \dfrac{\%\text{ Sewered} + \%\text{Nat. Chan. Elim.}}{2 \times 100}(1.70-0.42)$

2. Determine total point precipitation for time T_c and frequencies of interest from Fig. 2. Adjust for size of drainage area if necessary.

3. Determine percent of runoff:
   a. For natural area.
   b. Adjust for percent of area that is impervious.
      Normal Range
      Residential, 10 to 60 %
      Industrial, 40 to 70 %
      Commercial, 60 to 100 %

4. Compute total direct runoff in inches and divide by T_c to obtain I_r.

5. Compute Q_p.

Natural Areas Overland Flow, pavement Sewered Areas	$= 0.77\ A\ I_r$	Area in acres
	$= 500\ A\ I_r$	Area in sq. miles
	$q_p\ T_c = 570$	
Overland Flow, Turf	$= 0.44\ A\ I_r$	Area in acres
	$= 280\ A\ I_r$	Area in sq. miles
	$q_p\ T_c = 320$	

6. For Storm Drainage Design

Sample Tabulation

Column Number	Heading	Use
1	Location	Identification of location and section
2	Line	of drain.
3	Invert Elev., feet	Basic data for computation of slope, etc.
4	Length, feet	" " " " " " "
5	Slope, s, in %	Basic data and computation of equivalent length, leading to evaluation of T_c.
6	Friction Factor, n	
7	Equivalent Length, L'	"
8	Summation L'	"
9	(Summation L)$^{0.6}$	"
10	Time of Concentration, T_c plus Inlet Time	Computation of $T_c = 0.15\ (\Sigma L')^{0.6}$ and addition of Inlet Time.
11	Incremental Area	Tabulation of data to arrive at a value of weighted Imperviousness. Can be eliminated if approximation is permissable.
12	Incremental Area Imperviousness	
13	Incremental Area x Imperviousness	"
14	Accumulated Area x Imperviousness	"

15	Accumulated Area	Approximation is permissable
16	Weighted Imperviousness	Column 14/Column 15
17	$0.77\ I_r$	Obtained from curves such as Fig. 4 of Appendix I for appropriate frequency and imperviousness.
18	Discharge, Q_p, in cfs.	Design discharge equals column 15 x column 17.
19	Conduit	List shape, size and type of conduit.

APPENDIX II - NOTATION

A Area in acres or square miles

C_t Coefficient, hours/mile or minutes per foot, in $T_c = C_t L'^{0.6}$

I_r Average rate of direct runoff for time T_c, in inches per hour

K_d Reduction factor to be applied to instantaneous unit hydrograph peak to adjust for effect of an effective rainfall duration equal to T_c

K_s Adjustment factor to be applied to standard value of $q_p\ T_c = 1.03$ for values of T_s/T_c and L_{ca}/L other than 0.5

K Combined adjustment factor equal to $1.03\ K_d\ K_s$ or $658\ K_d\ K_s$

L Length of principal drainage channel, feet or miles

L' Length of equivalent channel with standard slope of 1% and friction factor of 0.10

n Friction factor, generally that used in Manning equation

q_i Peak of area-shape (discharge) diagram for one inch of runoff per unit area (1 acre or 1 square mile) of drainage basin

q_p Peak of instantaneous unit hydrograph (per acre or square mile) obtained by routing area-shape (discharge) diagram through reservoir type storage, T_s

Q_p Peak discharge of runoff hydrograph for total basin and total direct run-off volume spread over T_c hours

ΔQ Small increment of discharge

ΔS Corresponding increment of storage

s Weighted slope of principal channel in %, feet per hundred feet, and equal to the mean height of the channel divided by one half the length

T_c Time of concentration in hours or minutes and equal to time from end of rainfall to inflection point on recession side of hydrograph

T_s Index of reservoir type storage and equal to $\Delta S/\Delta Q$.

M Serial number of flood from arrangement of annual floods in order of decreasing magnitude

N Total number of years of record

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DIVINING RODS VERSUS HYDROLOGIC DATA AND RESEARCH

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The history of the basic-data programs are reviewed. The divining rod as a mystical hydrologic decision-maker is contrasted with modern basic-data programs. The extent and kinds of gaps in basic data are discussed with major emphasis on the deficiencies in research and interpretation of data.

Water problems, such as recurring floods and droughts, water surpluses and shortages, are everyday news. To solve these problems and to provide information for the sound and intelligent development of water resources, basic-data programs that are large in extent and scope have been established. How well are these programs doing their job?

Basic-data programs, like so many other things of the 20th century, are relatively new to the American scene. Much of the history of man reveals that lack of information has led him into enterprises that proved fruitful and that it has also wasted much effort. Although Columbus was careful to marshal all facts available, he was encouraged to try his westward trip by two errors, an underestimate of the size of the globe and an overestimate of the eastward extension of Asia. Ignorance of a different sort led those who followed him. Indeed, the North American continent was opened by conquistadores and colonists lured by fancies, not facts, and dreams, not data, even though increasingly accurate reports of what lay along the westward trails became available to those who would read them. In our West, fur traders, explorers, and later, scientists, reported what they saw. Yet ideas about the West remained fanciful, partly because the area was indeed fabulous, but also because the mind of the Nation was receiptive to the romantic idea. The mistaken hope that "the rain follows the plough" was a much more powerful influence behind the settlement of the subhumid Great Plains, than J. W. Powell's reports on rainfall

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published as early as the 1870's. Powell warned of the hazards of agriculture in areas that are neither humid nor arid and forecast the boom and bust that have marked agriculture on the subhumid-semiarid Plains.

The year 1879 is a key date in our progress. It was then that John F. Stevens, a railroad surveyor, discovered Marias Pass, in Montana, the last unknown mountain pass across the Rockies, thus, in a sense, marking the close of an era of geographic exploration in the United States. The year 1879 also saw the founding of the Geological Survey and the soon-to-follow hydrologic investigations as we know them today. With the waning of the era of exploration, basic data acquired a new meaning. The country now had to develop within fixed boundaries. In an earlier stage, action, whether based on fact or fancy, was the important thing; now, in a world of decreasing choices, decisions must be more sharply drawn.

In the meantime, the hydrologic frontier has been passed. Cheap water is no longer abundant everywhere. Projects for the control and use of water are now problems of national and even international concern. Sooner or later in planning a water project, the point is reached where a decision must be made: "drill here", "make it so many acre-feet", or "install so many kilowatts" according to the kind of project. In our planning we can use arbitrary methods and thereby short-circuit all needs for basic facts, we can resort to the opposite extreme of massive data gathering so as to provide minute details for every problem, or we can balance a moderate collection of facts with research and interpretation.

Man's early resort to magic, however, is still with us. Of all the hydrologic artifices, the one most commonly used is a forked stick of willow, ash, peach, or witch hazel. Such a forked stick, where used in the search for water, is called a divining rod or "witching stick". It is more widely known and used than the hydraulic engineer's current meter. The divining rod is believed by many to be capable of indicating the presence of water under ground. The user grips a branch of the fork in each hand and walks about with the point out in front. When he passes over an "underground river" or a "ground-water vein", the point is suddenly and inexplicably jerked down, pointing at a spot on the ground. "Here", he tells you, "one will find water." Why does this legacy of magic lore from the dark corners of the past persist alongside and despite modern technology? Or can we be scientific about witchery?

Witching for water is prevalent throughout the world because, rightly or wrongly, it gives an answer to many who seek water but who have not yet been reached by the hydrologic-data programs. Less than 10 per cent of our own country has yet been mapped or thoroughly explored for ground water; even where such work has been done, the reports and maps seldom reach all those who need them. Water in the ground is out of sight; therefore, to find water at any particular spot where the farmer may wish to dig a well may appear a highly uncertain business to the uninitiated. Like the medicine man, the water dowser gives a direct and unqualified answer depending only on the dip of a stock. This is an old trade, with many devotees. The literature dealing with it is large and ever growing (Ellis, 1917; Roberts, 1951; Sowder, 1955.)

The dowser gives a specific answer to allay the farmer's anxiety about his water supply and to enable him to keep his mind on the hard job of farming. Vogt (1953) suggests that water dowsing persists because it offers a short cut to a decision about a difficult feature in an uncertain environment. In many places random drilling has a good chance for success if only a domestic supply is needed. The geologist's advice to drill on a 50-50 chance might be

rejected whereas on the authority of a forked stick much useful ground water s developed.

Modern hydrology offers no support of water witchery. A lack of principles whereby experience can add to knowledge condemns divining as an acceptable hydrologic tool. The dowser's efforts to excuse "dry" holes supply a true measure of the trade. It is here that the dowser demonstrates his professional achievement. The driller is generally the chief target. Here are some of the excuses: "The driller busted the structure of the vein," "The driller moved the stake an inch," "The hole was drilled crookedly," or "My knife must have deflected the rod on that day." This kind of fare must compete with experimental science. A drilled hole, wet or dry, adds to the body of knowledge of the geologic structure and sharpens the ability of the geologist to make other recommendations in the region. The important job of the ground-water hydrologist a generation or more ago was to find water. The task today, as the late O. E. Meinzer put it, is much greater; it includes also the determination of availability, quantity, and quality of ground water. The most severe indictment of water dowsing is its harmful effect on efforts to obtain dependable information about the occurrence of ground water in an area.

Divining rods represent a class of techniques for reaching decisions painlessly but with little regard for factual substance and none for proven scientific principles. By way of contrast, we shall explore an opposite extreme represented by the idea that proper answers will become directly evident if only we can collect enough data.

The medieval appeal to witchery and dogma led Sir Francis Bacon in the late 1500's to reason that truth could be determined only through observation of nature. It was his further idea, in part, that if one only makes a sufficient number of observations, the truth will thereby become evident. This philosophy was a useful antidote to the divining-rod method of decision, but we have learned that, carried to extreme, data collection can be equally sterile. In the divining rod's use there is lack of both data and principles; data collection can lead to a surfeit of information to the neglect of principles. Both situations have historical and present interest for us.

Ever since Bacon's writings about the scientific method, scientists have concerned themselves about the relations between observation and conclusions. One of the first to analyze this problem in the modern sense was Augustus de Morgan, who, in 1872, write, "Modern discoveries have not been made by large collections of fact, with subsequent discussion, separation, and resulting deduction of a truth thus rendered perceptible." At the Mid-Century Conference on Resources for the Future, Neff (1953) stated that "We are pouring too much of our resources into the mass collection of data, without devoting sufficient resources to the development of hypothesis essential to the collection and the use of data presently available." Neff and De Morgan both claimed in essence that a special objective of data collection is the test of scientific hypotheses. By way of criticism, De Morgan cited the Greenwich Observatory, which was established in 1675 "to observe, observe, observe away at the moon, until her motions were known sufficiently well to render her useful in guiding the seaman." The discovery of the general law of gravitation, which explained not only the lunar motions but those of all other planetary bodies as well, was not made at Greenwich. Newton needed only 20 or 30 observations to prove his generalized theory. Had it not been for Newton, De Morgan, added, the whole dynasty of Greenwich astronomers might have worked away at 'nightly observations and daily reduction without any remarkable result."

De Morgan did not mean that only a genius could make significant contributions to general knowledge, but he used this example because the issues were clear cut. His point was that principles rank ahead of data. To emphasize further the importance of hypotheses, De Morgan added, "Wrong hypotheses, rightly worked from, have produced more useful results than unguided observation."

De Morgan was especially critical of the proposed enlargement of meteorological observations. He quoted Airy, who, in 1867, had said "Whether the effect of this movement will be that millions of useless observations will be added to the millions that already exist, or whether something may be expected to result which will lead to a meteorological theory, I cannot hazard a conjecture." Yet we know today that many meteorologic principles and forecast tools were derived and tested from these data, just as Kepler deduced his three laws of planetary motion from untiring analysis of the great mass of data compiled during a lifetime of observations by Brahe.

The modern view is that both the collection of data and their interpretation are necessary for reaching wise decisions. The problem is the proper division of effort. If data are too few to evaluate known principles, the designer of engineering works must choose between delaying the works in order to collect the data or to resort to extrapolation—what is facetiously called in the trade "galloping transpositions." On the other hand, data become too many when their collection is the main object and sufficient tools are not available for putting them to use—like the watchers of the "dickey" bird who, as Aldo Leopold (1953) put it, expend their efforts "labeling species and amassing facts about food habits without interpreting them."

Today we have extensive programs for the collection of hydrologic and related meteorologic data. A recent report (Subcommittee on Hydrology, 1956) showed that every Federal agency concerned with water is a source of hydrologic data, running through 30 items from consumptive use of water through precipitation, soil moisture, ground water, and streamflow to water waste. Agencies such as the Corps of Engineers or the Bureau of Reclamation collect information mainly related to their chief activities, flood control and irrigation. The Tennessee Valley Authority is active in collecting data virtually throughout the hydrologic cycle within its territory. The Weather Bureau and the Geological Survey are the only agencies that collect countrywide hydrologic information for public use as one of their major activities. These agencies collect the bulk of the available hydrologic data—precipitation, streamflow, ground water, and chemical and physical properties of water.

The governmental arrangement for data collection works well, for coordination is the rule. Rarely is there duplication of effort because there is so much to be done that there is little occasion for one man to duplicate the work of another. Yet, in any particular field of hydrology there is enough talent in two or more different agencies to assure that each man will stay on his toes lest his opposite numbers steal a march on him and make a contribution to the science in the specialty that he thought was his own. This healthy competition, constructively directed by informal coordination of the work of all the agencies and by the integrity of hydrologists throughout the Government, has made the level of achievement high and the volume of basic data large in spite of inadequacies of personnel and funds.

Nevertheless, a new water project nearly always reveals a deficiency. Some of these deficiencies are in the class of information. For example, the growth of the highway system and the upstream flood-control programs showed up a deficiency in information about the flow of small streams.

Heretofore, the emphasis in stream gaging has been on large streams, suitable mainly for power, navigation, irrigation, or water supply, neglecting the myriad small streams that are now of such great national concern in the Watershed Protection Program enacted in 1953.

The deficiency may be in coverage—for example, information on ground water and on the sediment carried by streams. Ground-water investigations have been carried out mainly where overdevelopment has created a crisis. Similarly, data on sediment loads have been collected mainly in areas of concern to Federal agencies planning large dams. Broad areas, such as New England, the Gulf region, the Great Basin, and the Pacific Coast are virtually without such records.

Most deficiencies exist because the basic-data programs are built chiefly around specific needs. The programs are directed mainly toward a narrow professional audience that is concerned with water development and control—chiefly those in governmental bureaus (Federal, State, and local). But hydrologic decisions are made in a wider sphere. Consider the builder or prospective homeowner who erects a house on the flood plain, without any recognition of perhaps the largest bulk of hydrologic data—those on floods. Or consider the farmer who had his well "witched" and who finds it inadequate, or who sees his farm pond washed out or filled with silt. The major faults, troubles, and failures of today often are not caused by a lack of basic data but to their disuse. Maximum use of these data can be fostered by their interpretation and putting the results into simple guides and digests for public use.

What proportion of the total investigative effort is devoted to the interpretation of this fund of data cannot be estimated, but it is undoubtedly less than 10 per cent. About 20,000 pages of water data are published annually. Only a few hundred pages are devoted to interpretive and research results. Not that tools are lacking. Rainfall, for example, is recorded as the events occur. On the other hand, the designer of a storm sewer may need to know the largest rainfall he can expect in a 1-hour period once in 20 years. These facts are found in the original record. Tabulation, rearrangement, and fitting the ordered data to a statistical "model" represents efficient use and interpretation of the raw facts. Other interpretations can be made from simple charts such as a map of mean annual yield of rivers or from complex prognostic weather charts. Interpretations may be aimed at specific projects, or at education. For some uses, as in water management, a flow record gives significant results immediately. For the development of a ground-water reservoir, the interpretations of well logs, pumping tests, water levels, and well spacings require highly involved and elaborate computations. Interpretations of a different kind serve to explain the nature of the occurrence and behavior of water to the general public, and to students of all ages who wish to enjoy a cultural appreciation of nature. The more detail that is available, and the higher the grade of the data, the wider is the range of interpretation that can be made and the more accurate are the applications.

There are other ways in which water data must be used. In a democratic society we all share in making decisions about water developments. Proposals for this dam or that irrigation scheme engender wide public discussion, much of which is not technically well founded. For example, the big dam-little dam controversy generates much heat but little light. Wise and proper decisions can be assured only if the public has direct access to facts in a form that they can understand. Raw tabulations of statistics of rainfall, streamflow, and ground water, necessary as they are, will not meed the need.

They cannot be used by the public until put into a form that bears on problems of public concern. These are proper goals of the basic-data program. The demand for specific data by governmental "action" agencies and by private groups causes a lag in interpretation of data. This demand has been so great that the data-gathering agencies have had little time or opportunity for a thorough examination.

The job is made more difficult because water problems change in emphasis. For example, the development of the deep-well pump and increases in agricultural yields and industrial demands have combined to make increasing demands upon ground water. Yet without reexamination and interpretation, the collection of basic data may become so inflexible and sterile that the programs cannot readily be adapted to meet changing needs.

Are we sure that we are not building water projects on inadequate data, or building homes on flood plains in disregard of known facts? Water needs and problems are as broad as the whole modern economy because water is an essential ingredient of our industrial might and of our hopes and plans for the future. Water facts, scientifically interpreted, can tie our plans to the realities of Nature. All of us can recognize a divining rod in the form of a forked stick, but it takes a patient skeptic to detect its more subtle counterparts.

We have seen that early appeals for collection of basic data by massive attack as a sole means for solving problems were a matter of concern as early as 1867. It must be clear that the usefulness of extensive programs for the collection of basic data depends on correct and timely interpretation of the data. The matter appears to be one of emphasis. An equally eloquent plea can be made both for more analyses of principles and for more quantitative data. A basic-data program can go wrong when there is an imbalance on either side.

REFERENCES

1. De Morgan, Augustus, 1872, A Budget of Paradoxes, London. Republished Open Court Publishing Co., Chicago, 1915.
2. Ellis, Arthur J., 1917. The Divining Rod: A History of Water Witching: U. S. Geol. Survey Water-Supply Paper 416.
3. Leopold, Aldo, 1953. Round River, Oxford University Press.
4. Neff, Philip, 1953. The Nation Looks At Its Resources: Rept. of the Mid-Century Conference on Resources for the Future, p. 311.
5. Roberts, Kenneth, 1951. Henry Gross and His Dowsing Rod: Doubleday.
6. Sowder, A. M., 1955. The Age-Old Debate About A Forked Stick: U. S. Dept. of Agri., Water: 1955 Yearbook of Agriculture, pp. 118-119.
7. Subcommittee on Hydrology, 1956. Principal Federal Sources of Hydrologic Data, Notes on Hydrologic Activities, Bulletin 8.
8. Vogt, Evon Z., 1953. Water Witching: An Interpretation of a Ritual Pattern in a Rural American Community: Sci. Monthly 75 (3): pp. 175-186.

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NUMERICAL SOLUTION OF FLOW PROBLEMS IN RIVERS

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(Proc. Paper 1810)

1. Outline and Summary

The purpose of this paper is to describe recent new methods which have been devised for dealing numerically with flow problems in large rivers and reservoirs for lengthy periods of time of the order of weeks. The basic idea of the method is to make use of the differential equations which formulate the well-known physical laws governing such flows as a basis for numerical calculation of future river depths and velocities. Such a method has become possible in a practical way only because of the relatively recent development of modern high speed digital computers, and of adequate methods for numerical analysis of the differential equations, since the amount of numerical computation necessary to solve the equations in question is very great indeed.

The idea of using the differential equations expressing the laws of conservation of mass and momentum as a means for treating flows in open channels is not at all new. In fact, it goes back to Massau(3)[1] as long ago as 1889. Since then the idea has been taken up by many others (for the most part in ignorance of the work of Massau) for example, by Preiswerk,(4) Thomas,(8) Craya,(1) Stoker.(6) Thomas, in particular, attacked the flood-routing problem in his noteworthy and pioneering paper(8) and outlined some numerical procedures for its solution based on the idea of using the method of finite differences. However, some of his methods would not necessarily furnish a good approximation to the desired solutions even if a large number of divisions of the river into sections were to be taken. In general, the amount of numerical work to be done looked far too formidable to be practical until rather recently.

During the past three years the feasibility of solving numerically such problems concerning flows in rivers has been investigated at the Institute of Mathematical Sciences of New York University under a contract with the Corps of Engineers of the U. S. Army,[2] and in cooperation with its Ohio River

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Division located at Cincinnati, Ohio. In fact, the entire project was greatly facilitated by B. R. Gilcrest of the Ohio River Division. Without his constant interest, advice, and willingness to help in providing and analyzing the data— a crucial task— we would not have been able to carry out our work.

Three test cases were selected, with the object of comparing the observed flood stages with those computed numerically:

1. The flood of 1945 (one of the largest on record) in the 375-mile stretch of the Ohio River between Wheeling, W. Va., and Cincinnati, Ohio,
2. The flood of 1947 through the junction of the Ohio and Mississippi Rivers at Cairo, Ill., and
3. The flood of 1950 through Kentucky Reservoir, at the end of the TVA system of power and flood control dams, which extends from Kentucky Dam near the mouth of the Tennessee River to Pickwick Dam 184 miles upstream.

In each case the state of the river or river system was taken at the beginning of the flood period, i.e., the river stages and velocities were assumed known everywhere at some arbitrarily selected initial time. For subsequent times the inflow from tributaries and the local run-off in the main valley were taken from the actual records, and the progress of the flood in the main stream was then computed for a future period of the order of weeks. In our cases the Univac digital computer, built by Sperry Rand Corporation, was used. The flood stages—that is, the heights of the river surface above sea level—as determined numerically, were then compared with those actually observed. This paper summarizes the methods used and the results of such calculations; for a detailed description the reader is referred to the reports of[2] by the authors.

The success of such an enterprise from a practical point of view depends on the answers to two questions:

1. Can the flood predictions be made accurate and quickly?
2. Is the cost of preparing and carrying out the numerical calculations reasonable?

The conclusions to be reported here are the result of a first effort which ought to be supplemented by still further investigations aimed at improving accuracy and on simplifying the methods, but there is no doubt that both questions can already be answered in the affirmative. We show, for example, in Fig. 1 the results of the calculations of river stages at the ends and in the middle of Kentucky Reservoir; as one sees the differences in calculated and observed stages over a period of three weeks are mostly to be measured in inches. The amount of computing time on the Univac required for calculating the flow over the three-week period was a little less than 4 hours. That such calculations can be made is, of course, a matter of considerable practical importance from various points of view. Once the basic data for a river, river system, or reservoir have been prepared and coded for a calculating machine it becomes possible to solve all sorts of problems quickly and inexpensively. For example, the effect on a flood wave of closing off a tributary by operating a dam, the relative merits of various schemes for serial operation of a system of dams, the preparation of tables showing the influence of changing the operating schedules of power dams at the ends of large reservoirs, or the solution of design problems in large reservoirs where, for example, navigation by barges is complicated by the occurrence of transient waves due to peaking

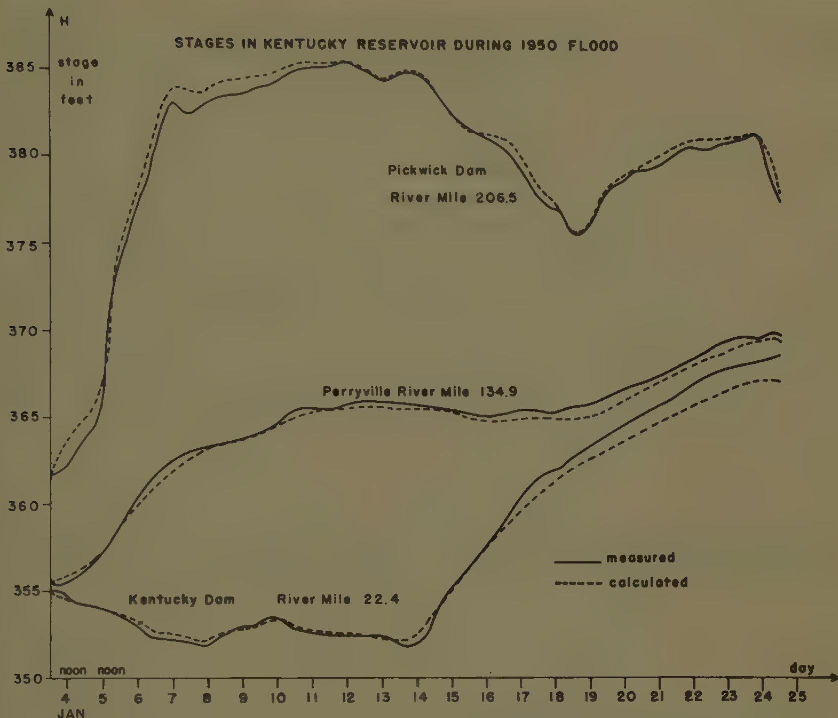

Fig. 1

operations of power plants, are all problems which can be attacked numerically.

These observations refer to the experience gained from the use of the Univac computer, which, fast though it is, nevertheless requires roughly a minute to compute the flow for an hour in the 375-mile stretch of the upper Ohio River, and about one-half minute for a flow of one hour in each of the other two cases under discussion here; thus for a flow calculation of 13-1/2 days in the Ohio River about 6-3/4 hours of Univac computing time were needed. However, if the much faster operating IBM-704 machine were used, these times (which, it should be repeated, resulted on the very first attempts to use numerical integration methods) would be cut down in the ratio of something like 1 to 15—thus reducing the calculating time needed for a flow lasting for weeks from hours down to minutes. Besides, calculating equipment is constantly being improved and made more available. It is thus perhaps not too visionary to predict that one day—and not too far distant a day either—the data for the whole of the Mississippi Valley river system will be coded for calculating machines, though not necessarily in just the form to be described here, ready to deal quickly and efficiently with problems of the most varied kinds relating to the system as a whole or to any part of it.

In discussing the usefulness and practicality of the numerical methods to be described here for dealing with large scale flow problems in rivers, it is of

course necessary to make comparisons with other methods. Hydraulic engineers have methods, termed flood-routing, which are relatively simple and reasonably accurate for dealing with some flow problems. Flood-routing is not always accurate, it is very tedious at a major junction like that of the Ohio and Mississippi Rivers, and it fails entirely in the case of a large reservoir such as Kentucky Reservoir. In addition, the flood-routing method requires a good deal of man power to carry out, which is at a premium in the Corps of Engineers, and probably also in the Weather Bureau, which is charged with the task of issuing flood predictions. Another method used to study flood problems is the method of hydraulic models, which have been built by the Corps of Engineers for some portions of the Mississippi valley. (They are located at the Waterways Experiment Station at Jackson, Miss.) These models are constructed on a large scale (covering acres in some cases). They are not true scale models since one of the essential factors in the problem, the roughness of the river bed, cannot be scaled, but must be dealt with empirically; such models are therefore really so-called analogue computers. More will be said later on this point.

2. Description of the Numerical Methods

The basic laws for flow in open channels are formulated in the following differential equations, in which subscripts denote differentiation with respect to the distance x along the channel and the time t:.

$$BH_t + (AV)_x = q \quad ,$$

$$V_t + VV_x + gH_x = -GV|V| \quad . \tag{2.1}$$

The first equation expresses the law of conservation of mass, the second the law of conservation of momentum. In these equations H, the stage, denotes the elevation of the water surface above sea level; V denotes the average velocity over a cross-section of the river; A denotes the cross-section area; B denotes the breadth of the river at the water surface; q denotes the volume of the inflow into the main channel per unit length of channel; and G represents the resistance coefficient.[3] The symbol $|V|$ means that the numerical value of V is to be taken: this insures that the term $GV|V|$ represents a resistance which is in the direction opposed to the velocity. The basic quantities to be determined, the stage H and velocity V, are both functions of the distance x along the channel and the time t. The coefficients A, B, and G are all functions of the distance x and stage H, while the quantity q characterizing inflows is a function of x and t. (It might be thought strange that the slope of the river does not occur in (2.1). However, it is really there implicitly, since $H_x = y_x - S$ in terms of the depth y of the river and the slope S of its bed.)

All of these coefficients, i.e. the quantities A(x, H), B(x, H), q (x, t), and G(x, H), must be known functions for the river or reservoir to be studied. One of the major tasks—and no small one, either—involved in solving the problems under discussion here is the determination of these coefficients from the basic data. The quantities A and B are purely geometrical and thus can in principle be fixed from topographic maps. The resistance coefficient G(x,H) was determined in our cases by using flow data from past floods in conjunction with the second of the equations (2.1); the terms V_t and VV_x, which are small compared with the other two in that equation, are neglected for this purpose

and thus G is determined from the relation $G = -gH_x/V|V|$. The quantity $q(x,t)$ characterizing inflows was known in our cases from the records, since we were testing our methods using observed values for past floods; in flood prediction problems it would be necessary to obtain this quantity from measurements at the mouths of the main tributaries together with estimates for the local run-off in the main valley. The details of the methods used by us for fixing coefficients are described and analyzed in the reports,[(2)] especially in Report III.

In addition to the differential equations, it is necessary to prescribe initial data and boundary data to have a completely formulated problem with a definitely determined solution. As initial data we must have, as is to be expected, the values of the stage H and velocity V at some instant of time chosen as the initial time at which calculations are to begin. Boundary data are required at the ends of the river stretch under consideration and can take different forms. The most reasonable way to prescribe boundary conditions in such a case as the upper Ohio River is to give the inflow in the main stream at the upper end (in our case, Wheeling) and to use a so-called rating curve, i.e. a relation between stage and velocity, at the lower end (Cincinnati) which is the result of observations of previous floods.[4] In Kentucky Reservoir, on the other hand, the appropriate boundary conditions to be imposed are the result of the special physical situation there, i.e. that the flow is controlled through releases of water at the dams closing the reservoir at both ends; in this case it is therefore the discharge rate of the water at the ends of the reservoir (or, equivalently, the velocity) which is prescribed for all times under consideration in the problem.

Once the initial data, inflows, and boundary conditions are given, the differential equations have a uniquely determined solution for the unknown quantities H and V for all future times. As a purely mathematical problem then, everything is in order. However, the differential equations are nonlinear and can not be solved by analytical means in general terms, quite apart from the fact that the coefficients are given empirically, and we are forced to turn to numerical methods. Even numerical methods were pretty much out of the question until recent years because of the intricacy and bulk of the numerical work necessary to solve such equations over hundreds of miles of a river and for periods of several weeks. Fortunately, the equations are exactly analogous to the differential equations of gas dynamics, and the methods and calculating equipment which were devised for solving such problems numerically during and subsequent to World War II are available for solving the flow problems under discussion here.

Thus the equations (2.1) for the determination of the unknown quantities $H(x,t)$ and $V(x,t)$ are sufficient to determine these unknowns uniquely once appropriate initial and boundary data have been prescribed. However, instead of trying to solve the two differential equations exactly for the unknown quantities for all values of x and t under consideration, one seeks instead good approximate values for H and V at a discrete net of points in an x, t-plane, as indicated in Fig. 2. For example, in the case of the upper Ohio River, the stretch from Wheeling to Cincinnati was divided up into 10-mile intervals Δx between net points, and approximate values of H and V were calculated at these points. (To have stages and velocities known at points ten miles apart is quite sufficient for practical purposes.) At the initial instant $t = 0$, i.e. on the x-axis itself, (which corresponded to Feb. 27, 1945 in this case), stage and velocity must be known at all net-points on the x-axis at this time. Once these

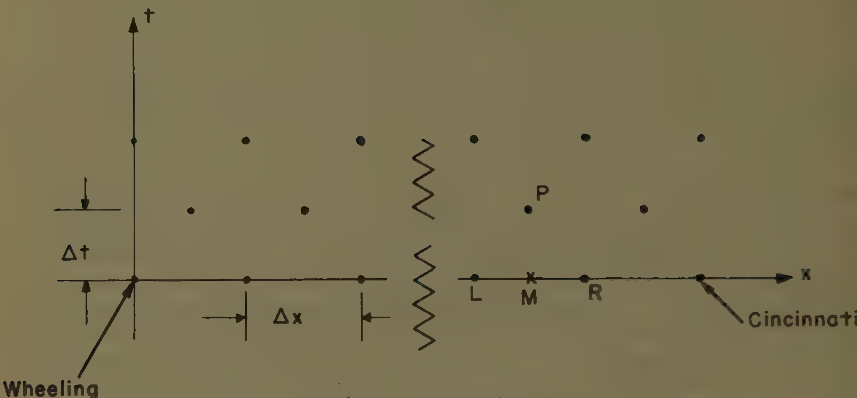

Fig. 2 Net in the x, t — plane

values are prescribed, and the inflows q are given (these are distributed to the nearest net points), the values of H and V are advanced by a time increment Δt through use of the differential equations. Technically, this is done by approximating derivatives in the differential equations by difference quotients which make use of values of H and V at net points only; the result is a system of algebraic equations which are solved by the calculating machine to yield approximate values of H and V at all net points. The values of H and V thus found will be good approximations to the correct values if Δx is small enough and if Δt is such that the ratio of Δt to Δx is under a certain bound (why this important condition is necessary will be explained later).

In all three cases presented here the interval between net-points was 10 miles, and the time interval Δt was taken to be 9 minutes. Thus we require data only at discrete points 10 miles apart, but such data must naturally represent a reasonable average over a ten-mile interval. Without going into details we indicate briefly, with reference to Fig. 2, how the differential equations (2.1) were replaced by finite difference equations and how the calculation proceeds, starting at the time $t = 0$, i.e. on the x-axis, when initial values for H and V are supposed known at all net points along the river. The method used to advance the solution to the next row of points distant Δt from the x-axis is then as follows. A staggered net is used, as indicated on Fig. 2. Consider the points L and R, where H and V are known; we give the procedure for determining their values at P, a time Δt later. First of all, values for H and V at M, midway between L and R, are computed as the averages of the values at L and R:

(2.2) $$H_M = \frac{1}{2}(H_R + H_L) \quad , \qquad V_M = \frac{1}{2}(V_R + V_L) \quad .$$

The time derivatives at M are then given approximately by

(2.3) $$H_t = \frac{H_P - H_M}{\Delta t} \quad , \qquad V_t = \frac{V_P - V_M}{\Delta t} \quad ,$$

and derivatives with respect to x at M are calculated similarly:

$$H_x = \frac{H_R - H_L}{\Delta x} , \qquad V_x = \frac{V_R - V_L}{\Delta x} . \tag{2.4}$$

These approximations to the derivatives at M are inserted for the corresponding derivatives in the differential equations (2.1), the coefficients being evaluated at point M. The result is a pair of algebraic equations which can be solved to yield values for H_P and V_P, as follows:

$$\begin{aligned} V_P &= \tfrac{1}{2}[V_R+V_L] - \frac{\Delta t}{\Delta x}\left[\frac{V_R^2-V_L^2}{2} + g(H_R-H_L)\right] - \frac{\Delta t}{2}[G_RV_R^2+G_LV_L^2] , \\ H_P &= \tfrac{1}{2}[H_R+H_L] + \frac{2\Delta t}{(B_R+B_L)}\left[q_{LR} + \frac{(A_LV_L - A_RV_R)}{\Delta x}\right]. \end{aligned} \tag{2.5}$$

Here q_{LR} is the volume of inflow per unit time between the points L and R. At the boundaries, a somewhat different procedure is used (see (2)). In analogous fashion the solution is advanced from one line of net-points parallel to the x-axis to the next line Δt minutes later. In principle, the idea is simple, but in practice a great deal of calculation is needed, since, for example, 1100 constants must be available to the calculating machine in the case of the Upper Ohio River to define the coefficients A, B, G, etc. In addition, since the solution is advanced only 9 minutes at a time the number of net points, and hence the number of values of H and V to be determined is very large for a stretch of river 375 miles long and for times of the order of weeks.

It is conceivable a priori that the amount of computation necessary might be so large as to make this scheme impractical. It is even conceivable that it might take longer to predict a flood numerically than the time of duration of the flood itself.[5] Fortunately, as has already been noted, reasonable calculating times resulted in the cases reported here.

The question of the accuracy of the approximation has been studied for differential equations of the type in question here. It is known that approximations to the exact solutions can be made with as much accuracy as desired by taking the mesh-widths in the finite difference scheme referred to in connection with Fig. 2 sufficiently small. However as was already mentioned, it is a striking and interesting fact that it is not possible to dispose of the size of the mesh width interval Δt in the time variable in an arbitrary way; instead, once an interval Δx has been chosen—presumably small enough for the accuracy desired in the approximate solution—the time interval Δt must not be taken larger than a certain multiple of Δx which depends upon the stage H and velocity V. If this rule is violated, the consequence is not simply an inaccuracy in the results; rather, totally unusable values for the unknowns H and V are obtained. This fact also has decisive practical importance, since the amount of calculating machine time needed to solve a given problem is directly proportional to the number of net points and hence inversely proportional to the permissible size Δt of the time increment. For example, in our three problems in which an interval $\Delta x = 10$ miles was chosen for the space variable (on the basis of preliminary calculations in simple cases) the maximum time interval permissible was about 10 minutes; we therefore chose 9 minutes as time increment to be on the safe side. Since some of the calculated flows had

a total time duration of three weeks, it is obvious that the limitation of 9 minutes as the time step by which the solution could be advanced at any one step means that a large number of net points was needed, with correspondingly large amounts of calculating machine time.

The limitation that must be imposed on the ratio of the time increment to the space increment is not simply the consequence of a purely mathematical fact having to do with the convergence of a numerical scheme to the correct solutions; rather, it stems from a fundamental physical property of the flow. We are concerned in these problems, as also in the analogous problems in gas dynamics, with a wave motion. A characteristic property of wave motions in a river (and in the analogous situations in gases and elastic solids also, for that matter) is that a small disturbance in the flow created over a limited portion of it—an influx of water from a tributary, say—will create a disturbance (called a wavelet) the front of which propagates both upstream and downstream at a rate, relative to the stream flow, which depends on the local value of the stage H; in fact the propagation speed of wavelets relative to the stream is given by $\sqrt{g\frac{A}{B}} = \sqrt{gd}$, with d the average depth of the water at a given cross-section. In other words, if we interpret this remark in the x, t-plane of Fig. 2, it means that a disturbance created in a certain region at a given time spreads out in a definite fashion and affects the flow only in definite regions of the x, t-plane. Or, upon reversing this consideration to put it in the way which is most important for present purposes, the magnitudes of the stage H and velocity V at any particular time and place, that is, at any given point of the x, t-plane, are determined from the values of these quantities at an earlier time only on a definite and uniquely determined segment along the river, and are totally uninfluenced by what happens at any other points along the river. This has a most important bearing on the problem of calculating a wave motion, as we explain with reference to Fig. 3. We suppose that calculations

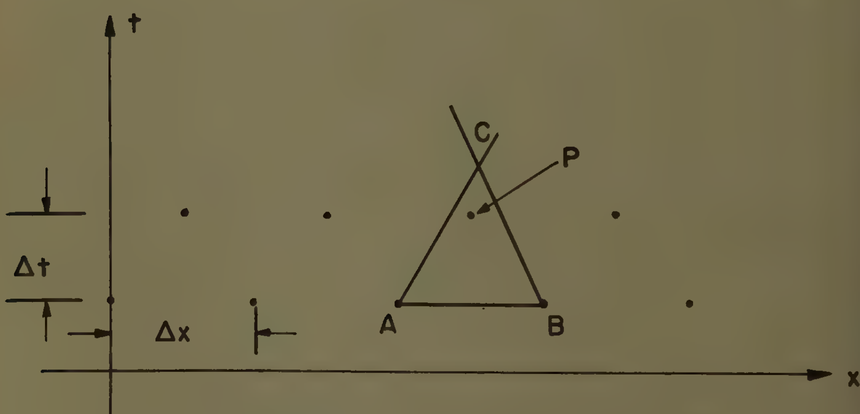

Fig. 3 Characteristics

for a flood wave had already been advanced to a certain time t, i.e. to a certain line in the x, t-plane parallel to the x-axis, and that it is desired to advance the solution to the next line of net points corresponding to a time Δt later. The new values at the point P, for example, are to be determined from the known values at the two nearest points A, B (at the time Δt earlier) through use of the differential equations expressed approximately in terms of difference equations. From our discussion of wavelets it seems clear that such a method would not work if point P were to fall outside of the region of the x, t-plane which is affected by conditions imposed over the segment AB, since if it did, one would not be using all of the relevant data that is required physically to fix the motion completely at point P. The curves AC and BC which define the region within which P must be chosen are called characteristics; they are curves fixed by the solution, which, in physical terms, are nothing but the loci along which the wave fronts of small disturbances propagate—in fact, the curve AC, for example, is a portion of that wave front caused by a small disturbance at A which moves to the right relative to the stream. Thus, the more rapid the propagation speed of wavelets the smaller is the permissible time increment Δt, since high propagation speeds mean that the wave fronts move along curves inclined at small angles to the x-axis. Thus numerical calculation of wave motions is made easier and more rapid if wave propagation speeds are small; this is the reason it was stated earlier that gas dynamics problems are in some respects more difficult to deal with than flood wave problems.

The flood wave motions in rivers are the result of the nonlinear superposition of elementary wave disturbances which move both upstream and downstream from the sources of disturbance—which are, it should be said, not caused solely by flows into the stream from tributaries, but also by variations along the river in cross section, in roughness, in operating conditions at dams, etc. Most engineering methods of flood routing are based on the fact that a flood wave in a river is commonly observed to progress downstream only, and with a velocity which can be estimated or fixed from data obtained from past floods; this observation, together with the use of the law of conservation of mass, leads to a relatively simple method for calculating the progress of the wave. At first sight, this would seem to be in crass contradiction with the analysis made above, which involves waves propagating upstream as well as downstream, particularly when, in addition, the propagation speed of a flood wave (defined, say, as the speed of travel of its crest) is actually slower in ordinary rivers by a factor of 1:4 or 1:5 than the propagation speed of wavelets. There is no real contradiction, however. What happens is that the resistance to the flow caused by the roughness of the river bed has the effect of slowing down the propagation speed of the main part of the wave, which nevertheless is the result of superposition of wavelets traveling in both directions. The engineering method thus could be expected to work best when the effect of propagation upstream is relatively small, and this is often the case for a flood in a long river. However, if there are many obstructions in the river, or if tributary inflows, so large as to cause important backwater effects occur (such as, for example, the flows at the junction of the Ohio and Mississippi Rivers) a method which ignores the finer details involved in the method employing wavelets may not be sufficiently accurate. The reason why the problem presented by flows in a large reservoir, such as Kentucky Reservoir, have never yielded to the conventional methods of solution is doubtlessly the fact that propagation of waves upstream due to reflection from the face of

the dam at the downstream end, and similarly at the upstream end, is a factor of major importance in such cases. In Kentucky Reservoir, for example, a wavelet propagates with such a speed as to traverse the length of the reservoir (a distance of 184 miles) in a time of the order of 1/4 day; for a flood calculation covering a period of three weeks there would thus be time for something like 80 such traverses of waves to be made back and forth between the dams at the two ends; and since the resultant effect at any one point comes about through a superposition of these waves it is not surprising that it is essential to consider wave propagation in both directions in this case.

3. Description of Results. Comparison with Observations

We proceed to give an account of the actual results obtained in the three cases we have treated using numerical methods.

We begin with the case of the upper Ohio River, shown diagrammatically in Fig. 4. Figs. 5 and 6 show calculated versus observed river stages (given as heights above sea level) at Maysville and Pomeroy for somewhat more than two weeks of the 1945 flood; this period includes the crest of the flood. As one sees, the agreement of the calculated with the observed stages is quite good, the differences between the two being mostly less than one foot. This case, which was the first one dealt with by us, is, however, the one for which

REACHES IN THE OHIO RIVER

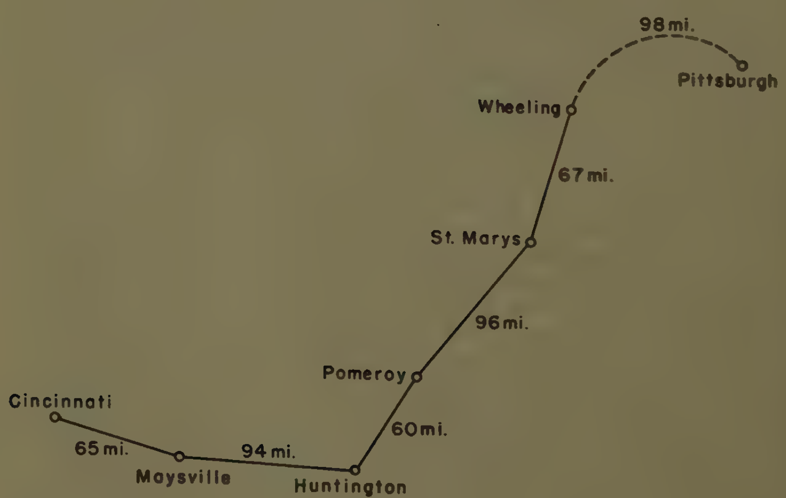

Fig. 4

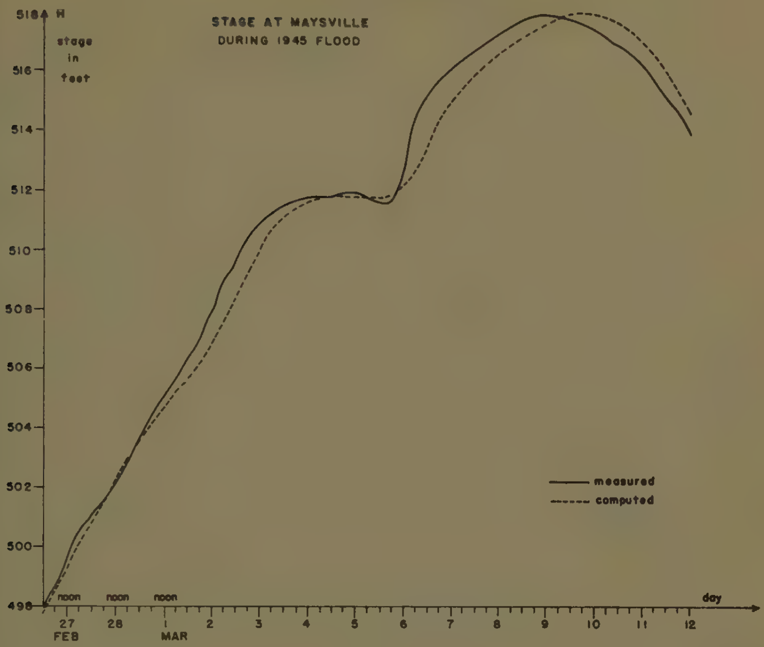

Fig. 5

our results are the least accurate, for reasons which will be discussed later on.

Fig. 7 shows a schematic diagram of the junction of the Ohio and Mississippi River. The flow velocities into the river at Thebes and Metropolis were given, and the stage-discharge relation was given at Hickman on the lower Mississippi. Figs. 8 and 9 show calculated and observed stages at Hickman and Cairo, and Fig. 10 the same things at Metropolis for a period of three weeks during the 1947 flood in the Ohio River. Again the agreement of the two is very good, and even minor variations in the curves for the observed stages are mirrored in the curves obtained by calculations. The differences between observed and calculated stages in this case are of the order of inches.

Fig. 11 is a diagrammatic sketch of Kentucky Reservoir at the mouth of the Tennessee River. In this case the motion of the water in the reservoir was almost entirely controlled by the releases of water at Kentucky Dam and Pickwick Dam at its ends, since inflows from the local drainage area were quite small. Fig. 12 is a graph showing the releases at these two dams during three weeks of the 1950 flood. As one sees, the releases varied quite rapidly at the beginning—to empty the dams in order to provide storage for the flood water coming down. The results of calculating stages at the two ends of the reservoir and at Perryville near its center have already been shown in Fig. 1. Again the agreement between observed and calculated stages is very good for the entire period of three weeks. It is worth repeating that the successful outcome of the calculations for Kentucky Reservoir is a fact of particular importance, since the conventional flood routing methods fail in such cases.

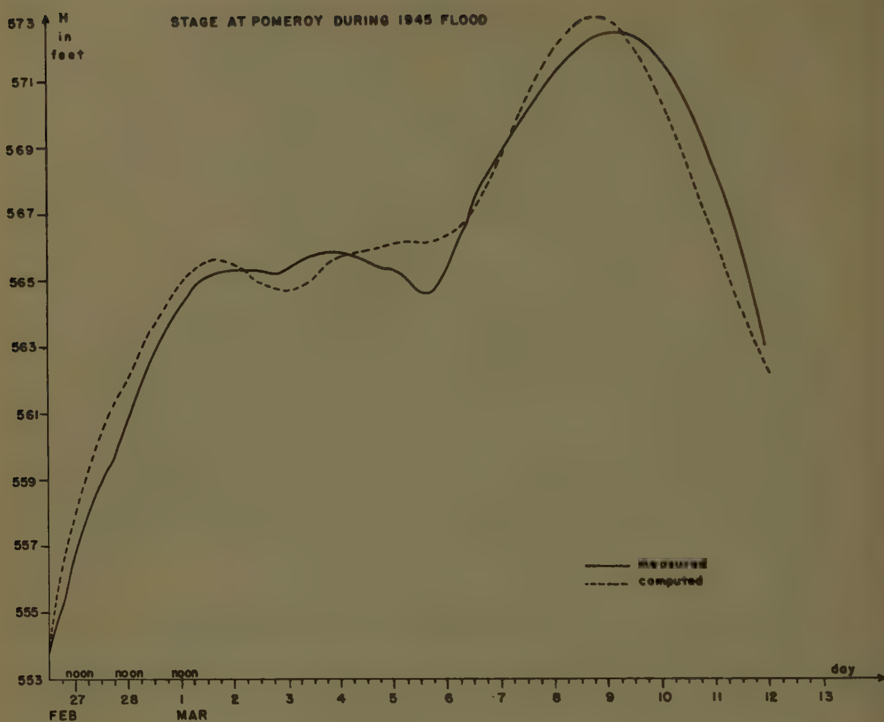

Fig. 6

It should be emphasized that the amount of Univac computing time required in these cases was one minute for determining the flow for an hour in the case of the Ohio, and half that in the other two cases. If the IBM-704 were to be used these times would be cut in the ratio of approximately 1:15.

4. Comparison with Model Experiments

It has already been mentioned that hydraulic models of rivers have been built at the Waterways Experiment Station in Jackson, Miss. for the purpose of studying flood waves, and that they are large—covering acres of ground. It is of interest to compare this method with the numerical method, for one thing because the two methods have much in common, and besides, a discussion of these facts makes it possible to touch upon a number of things that are of general interest.

The models are built with a much exaggerated vertical scale compared with the horizontal scale. The contours of the river valley are reproduced accurately in concrete. Specially designed pumps have been devised to reproduce given inflows. The river stages at the main gaging stations are continuously measured with delicate gauges and recorded remotely on graph paper in a special building which houses the electronic equipment. It has already been mentioned that one of the main physical features governing the flow in a

THE JUNCTION PROBLEM

Fig. 7

river, namely the roughness of the river bed, which causes a resistance to the flow, does not scale simply for a model. Instead it is necessary to introduce obstacles in the bed of the model in such a way that actually observed past floods are accurately reproduced; only after this has been done can the models be used to study floods. At Jackson, these obstacles take the form of brass knobs screwed into the bed of the model, and of wire screen.

The two methods being compared are closely analogous even in details. The purely geometrical quantities, such as cross-section areas and breadths must be available for both methods; in the numerical method, however, average values of these quantities over ten mile intervals have been proved to suffice, and only the area and breadth, not the exact contours, are needed. The resistance is finely adjusted in a model simply by experimentation: brass knobs, or wire screens, are added or removed until the flood stages check with those from actually observed past floods. In the numerical method a first estimate for the resistance is obtained from records of past floods. Nevertheless, we had expected that modifications of the resistance coefficient, just as in model studies, and perhaps also of the average cross sections, would be necessary in order to reproduce accurately a given flood over a lengthy period of time by means of a numerical calculation. However, it turned out that in only one of the three problems discussed here was a

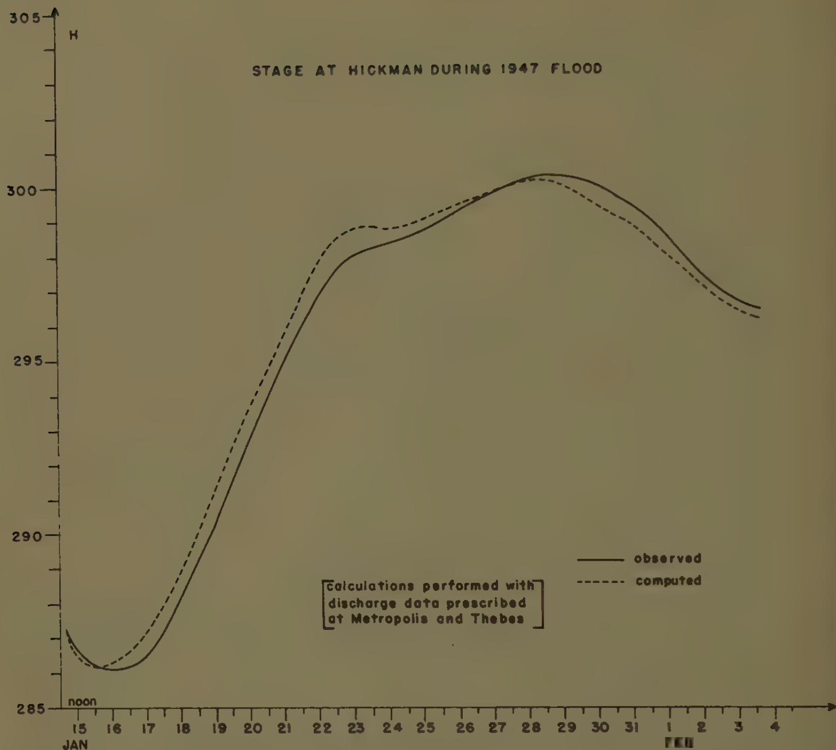

Fig. 8

substantial revision in the first estimates for cross section and resistance coefficients needed. For the junction problem and the problem of Kentucky Reservoir practically no changes from the values first obtained from the basic data for both cross-sections and resistance coefficients were needed to check the observed flows. For the problem of the upper Ohio River, however, considerable revisions, particularly of the average breadths, were necessary in order to reproduce the 1945 flood with the accuracy shown in Fig. 5. As a consequence, when calculations were made for the flood of 1948 in the Ohio, it was found that the results, though not bad at many places nevertheless were considerably less accurate—in the vicinity of Huntington, W. Va., particularly—for this flood than they were for the 1945 flood. The natural inference is that the basic original data available for the Ohio River were less accurate than they were for the junction problem and for Kentucky Reservoir. This was indeed the case, particularly with respect to cross section areas; in the case of the Ohio River these were determined from storage volumes obtained by balancing flows observed in past floods over the intervals, called reaches, between the gaging stations shown in Fig. 4, which have lengths of from 60 up to

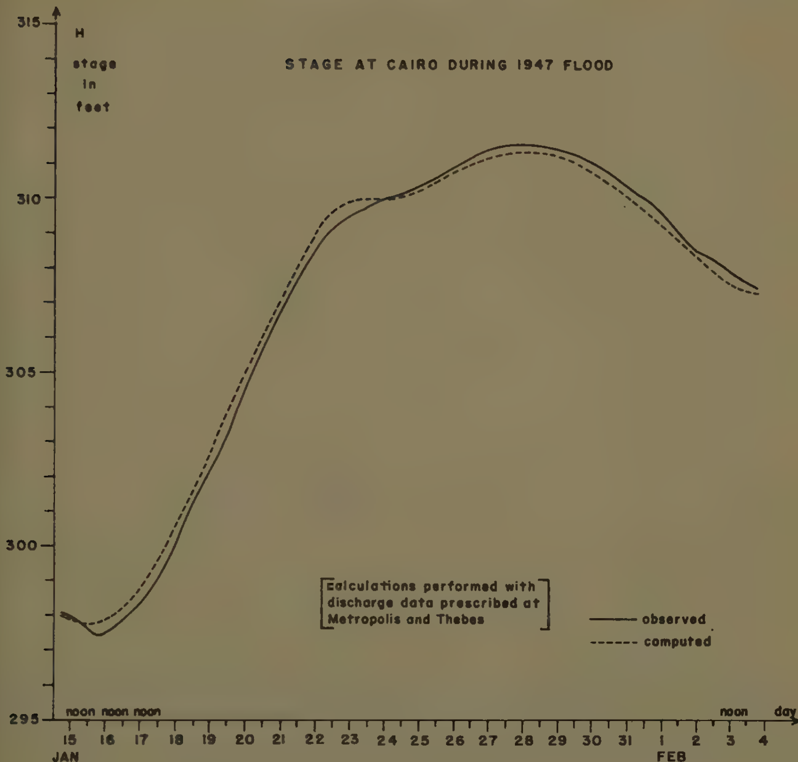

Fig. 9

nearly 100 miles. In the other two cases, accurate average cross sections were obtained from topographical survey data. The fact that fairly reasonable results for the Ohio River were nevertheless obtained indicates that it is possible to go rather far with the use of average properties over large distances; however, our coefficients for the Ohio River should be revised and made more accurate. The accurate results obtained in Kentucky Reservoir for the flood of 1950 were checked by making calculations for the flood of 1948, with good results; this was to be expected since the observed flood of 1950 was reproduced accurately by the calculations with practically no changes in the coefficients obtained from an analysis of the basic data. It is thus indicated that the method of numerical calculation will in general require no great revision of the coefficients if average cross sections are obtained from topographical surveys.

CONCLUSION

We have seen that the method of numerical computation is flexible. That is, it is simple to vary the physical parameters since all that is necessary is

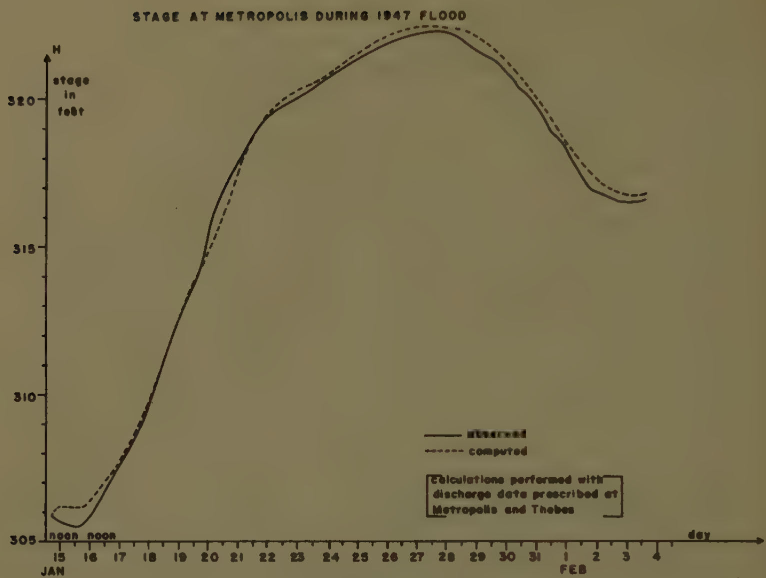

Fig. 10

to change some instructions to the calculating machine. For instance, even cross section areas might need some revision after comparisons of calculated with observed floods since the topographic surveys might be so old that the river channel had changed. Some rivers, such as the Missouri River for example, can change their characteristics fairly rapidly—because of scouring of the bed, or conversely by silting up, even to the extent that changes in the course of the river occur. Such changes would manifest themselves in discrepancies between observed and calculated river stages; revision of the basic data could then be made. In other words, constant back-checking of calculated with observed floods could lead to steady improvement in the accuracy of the basic physical data for a given river. Another feature in which numerical methods are flexible concerns the method of dealing with the local runoff and the flow from tributaries. In the numerical method there is no difficulty of any kind involved in feeding flows in at any net points whatever. Furthermore once the coding of the basic data for a river has been completed, the necessary instructions to the computing machine are placed on a few small magnetic tapes which are available at any moment to solve a given problem.

These remarks should not be regarded as adverse criticisms of model studies in hydraulics. For example, it would be difficult if not impossible to deal numerically with the details of the flow close to Kentucky Dam itself and through its turbines and over its spillways: for such things model studies are needed. Even for such a case as a large reservoir the method employing models was the only one available until the recent development of high speed digital computing machines, and current methods of numerical analysis, made possible methods of the sort described in this paper.

THE KENTUCKY RESERVOIR

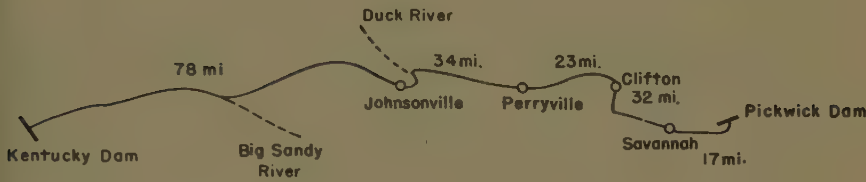

Fig. 11

FOOTNOTES

1. Numbers in parenthesis refer to the bibliography at the end of the paper.

2. Although this work was initiated by the office of the Chief of Engineers, U. S. Army and was administered by the Ohio River Division of the Corps of Engineers, the views expressed here are solely those of the writers and do not necessarily represent those of our sponsoring agency.

3. For a derivation of these well-known equations, see, for examples, references (5) and (7). Also, detailed descriptions of the methods described briefly in this paper can be found in the reports of the writers (2) and in (7).

4. This is what electrical engineers would call a matching impedance condition, the purpose of which is to replace appropriately the effect of the remainder of the river below Cincinnati.

5. In early attempts to make weather predictions numerically, such situations actually occurred.

BIBLIOGRAPHY

1. A. Craya, Calcul graphique des régimes variables dans les canaux, La Houille Blanche, 1946.

2. E. Isaacson, J. J. Stoker, A. Troesch, Numerical Solution of Flood Prediction and River Regulation Problems.
   Report I: Derivation of Basic Theory and Formulation of Numerical Methods of Attack, 1953.
   Report II: Numerical Solution of Flood Problems in Simplified Models of the Ohio River, 1954.
   Report III: Results of the Numerical Prediction of the 1945 and 1948 Floods in the Ohio River, of the 1947 Flood through the Junction of the Ohio and Mississippi Rivers, and of the Floods of 1950 and 1948 through Kentucky Reservoir, 1956.

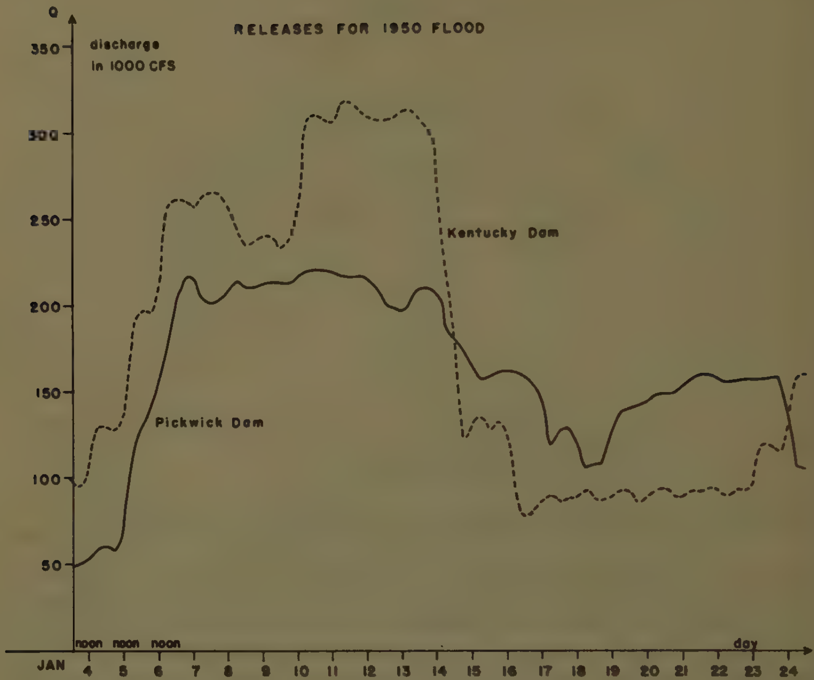

Fig. 12

Published as reports of the Institute of Mathematical Sciences at New York University under the terms of a contract with the U. S. Army, Corps of Engineers, Ohio River Division.

3. J. Massau, Mémoire sur l'intégration graphique des équations aux derivées partielles, Annales des ingénieurs sortis de Gand, Tome XII, 1889.

4. E. Preiswerk, Anwendung gasdynamischer Methoden auf Wasserströmungen mit freier Oberfläche, Mitteilungen aus dem Institut für Aerodynamik, Technische Hochschule, Zürich, 1938.

5. H. Rouse, Engineering Hydraulics, New York and London, 1950. (Especially the article by B. R. Gilcrest, p. 635.)

6. J. J. Stoker, The Formation of Breakers and Bores, Communications on Applied Mathematics, Vol. I, p. 1, 1948.

7. J. J. Stoker, Water Waves, Interscience Publishers, New York, 1957.

8. H. A. Thomas, The Hydraulics of Flood-Movements in Rivers, Carnegie Institute of Technology, Pittsburgh, Pennsylvania, 1937.

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QUEUING THEORY AND WATER STORAGE

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(Proc. Paper 1811)

ABSTRACT

A method for determining the amount of holdover storage for regulating streamflow is presented based on analogies with queues. Where discharge varies linearly with reservoir contents a unique expression is derived. The queuing analogy leads to a general solution by "probability routing". The effect of sequential correlation is also discussed.

Reservoir storage is usually calculated by the mass-curve analysis introduced by Rippl (1883). This method, or modifications of it, is based on a detection of periods of the part record when flows were less than required for a given need. The cumulative difference between desired flow and the actual flow during the period of flow deficiency equals the needed storage to supplement the available flow. The mass-curve method appears deficient to the extent that it yields unique answers that may be deceiving in accuracy. Reservoir design involves some chance that the final decision may prove too small or too large. These alternatives, although not often expressed in engineering reports, are implied in any water development that is linked to a variable resource like river discharge. There is need, therefore, for improved methods of storage design that give tangible expression to the probabilities of shortages and excesses. The theory of queues (waiting lines) offers a new approach to the determination of storage on a probability basis.

All of us have experience in waiting on line, and waiting offers a good time for thinking about the logic and operations of a queue as it is sometimes called. A line waiting for service at a ticket office, for example, is an obvious queue. There are other kinds of queues, more subtle in form, but quite similar in operation. Airplanes stacked over an airport, store inventories, persons waiting for a clear telephone line, all are queues. A dam placed

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across the river with gates or openings to control the outflow impounds water. The impounded water is a queue. The inflow to reservoir represents the arrivals, the regulated outflows represent the departures, and the impounded water represents the queue. The analysis of queues is a problem treated in the new science of "operations research." (Churchman, et al, pp. 389-471, 1957.) However, few of the developments in queuing theory, as it is called, are directly applicable to storage analysis. A paper by Moran (1954) recognized the analogy between queues and water storage and introduced probability theory in storage analysis. This paper is intended to develop the subject further.

One of the most useful applications of queuing theory is that it provides a classification of the factors that can be used to describe water storage.

1. Arrival rate. This is generally expressed by a frequency or time distribution of the arrivals, the demands for service, or in the case of reservoirs, the inflow.
2. Queue discipline: This is the rule establishing priority of servicing among those on the queue. Ordinarily the rule is first come, first served. But special rules may be set up or observed. Quick jobs on my desk get attention before long jobs. One acre-foot of water in a reservoir is usually as useful as another; but selected drafts may be made from high or low levels because of a desire to draw off water that is cold or warm, turbid or clear, or water differing in salinity.
3. Service function. This is the rule defining the rate of service of items in the queue. The service might be automatically controlled like a traffic light or a fixed reservoir outlet. Or the control might be manual like a traffic cop or a gate tender at a reservoir. In either case the control action is described by what is called the "service function". The traffic light controls the flow of traffic in time sequence; the reservoir orifice controls the outflow in proportion to the head and hence to the storage; the cop controls the flow of traffic in time sequence, but hopefully he gives weight to changes in relative density at the crossroads; the gate operator receives instructions from a dispatcher who takes account of demands and the volume of storage on hand and in prospect.
4. Attrition rate. The longer a line awaiting service, the greater is the likelihood that persons will either prematurely depart or refuse to join the queue. The attrition rate depends on the kind of service. People will wait longer for a doctor's attention than to buy a pack of cigarettes. There is a parallel with reservoirs. Here the loss is that due to evaporation which is also proportional to the amount of storage that exists at any time. The rate, of course, depends on the regional climate and the geometry of the reservoir. Decisions on tolerable queues or optimum size of reservoir must consider loss through attrition or evaporation.

The parallels appear sufficiently close to warrant consideration of this method of classification in the design of those storage reservoirs, built to store water for subsequent use. Reservoirs may be classified in different ways—capacity is one way, detention period (the ratio of capacity to average river flow) is another. In queuing theory, reservoirs are classified in the way we have just described queues—principally by the inflow distribution and the service function. The inflow distribution is generally expressed by the familiar duration curve, and queuing theory provides the long-sought means for

relating the duration curve to storage. Service functions are less formally defined in customary reservoir design. Some are depicted in Fig. 1. Type IA includes those where the outflow discharge is linearly related to storage; the reservoir is intended to maintain a minimum low flow b, and to decapitate floods. Type IB is the orifice type, and is typical of reservoirs used for flood control. Type II is the usual type of regulation assumed in most analyses of storage by the mass curve or Rippl diagram method (Rippl, 1883). Discharge is maintained at a uniform draft unless it is empty or full. The "uniform draft" may vary seasonally. In Type IIA, where the uniform draft is less than mean (low water regulation), capacity is established such that frequency of an empty reservoir is limited. In Type IIB, where the uniform draft is greater than mean (flood control), capacity is established such that frequency of uncontrolled spills are limited. Type III shows a seasonal type of rule curve. The

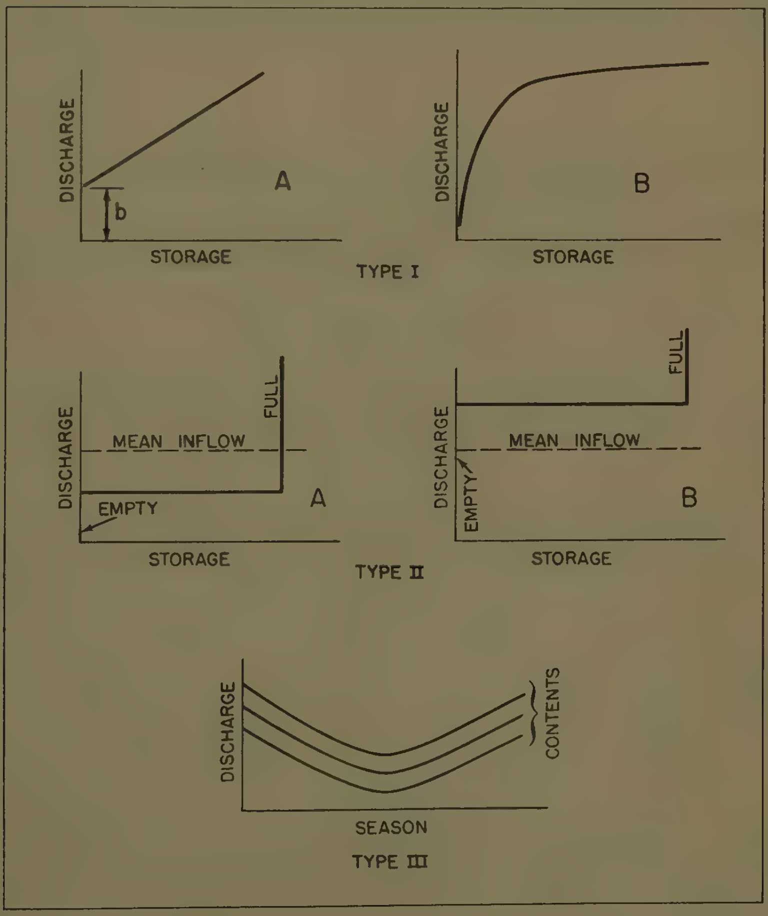

Figure 1.--Three types of service functions.

discharge varies seasonally and with contents. All of these general classes and combinations of them can be handled by probability analysis. The principal technical problems of hydrologic design of a reservoir are usually met by determination of the frequency with which a proposed reservoir will contain differing amounts of water (called the frequency distribution of storage). Other factors of interest are the frequency of spill, the frequency that the reservoir may be empty, and the frequency distribution of reservoir discharges. Some of these applications are developed in this paper.

Two kinds of solutions are given. The first example is an algebraic solution applicable only to a linear service function, such as Type IA. This special solution is of interest because of the inferences regarding storage that may be drawn from it. However, direct algebraic solutions must necessarily be restricted because some service functions or arrival rates become cumbersome when expressed algebraically, a more flexible method, termed "probability routing" is given in the second part of the paper.

In all the examples to follow, storage is given in units equal to flow in one unit of time. Thus, if flow is in acre-feet per year, storage is acre-feet; if flow units are in cfs and monthly units are used, then storage is in cfs-months.

Example 1

Service Function: $D = b + kS$ in which D is the outflow discharge and S the storage. (1)

A change in storage (length of queue) occurs whenever there is a difference between rates of inflow (arrival) and discharge (departures), hence

$$\Delta S = I - D \qquad \text{in which I is arrival rate (inflow)} \tag{2}$$

from (1) and (2) and Fig. 1 (Type IA function)

$$\Delta D/k = I - D$$

If time intervals are sufficiently close

$$D_2 - D_1 = kI_2 - kD_2 \tag{3}$$

where subscripts 1 and 2 refer to successive time intervals.

From Eq. (3) we obtain

$$D_2 = \frac{k}{1+k} I_2 + \frac{1}{1+k} D_1 \tag{4}$$

We may also write

$$D_3 = \frac{k}{1+k} I_3 + \frac{1}{1+k} D_2 \tag{5a}$$

$$D_4 = \frac{k}{1+k} I_4 + \frac{1}{1+k} D_3 \tag{5b}$$

$$D_5 = \frac{k}{1+k} I_5 + \frac{1}{1+k} D_4 \tag{5c}$$

etc.

Inserting the value for D from Eq. (4) into Eq. (5a) and for D from Eq. (5a) into (5b), etc. we obtain

$$D_n = \frac{k}{1+k}\left[I_n + \frac{1}{1+k} I_{n-1} + \frac{1}{(1+k)^2} I_{n-2} + \frac{1}{(1+k)^3} I_{n-3} + \cdots\right]$$

Hence, each discharge (departure) is a weighted average of the current and preceding inflows (arrivals). Since each outflow discharge is made up of successive proportions of the inflows, the standard deviation of discharges, σ_d, can be computed from the sum of the squares of the standard deviations of the components, hence:

$$\sigma_d = \sqrt{\left(\frac{k}{1+k}\sigma\right)^2 + \left(\frac{k}{(1+k)^2}\sigma\right)^2 + \left(\frac{k}{(1+k)^3}\sigma\right)^2 + \cdots}$$

$$\sigma_d = \sigma\sqrt{\frac{k}{2+k}} \tag{6}$$

where σ is the standard deviation of the inflows.

The distribution of the discharges will therefore be defined by the mean $\bar{X}$ which would necessarily be identical with the mean inflow and a standard deviation σ_d.

Storage Capacity Required

The average capacity follows immediately from the service function:

$$D = b + kS \tag{1}$$

Since this is a linear relationship, we can insert means and solve:

$$\bar{S} = \frac{\bar{X} - b}{k} \tag{7}$$

where $\bar{X}$ is the mean rate of inflow, and $\bar{S}$ is mean storage.

But greater capacity than average is required, otherwise the reservoir would frequently spill uncontrolled. We can set capacity equal to any desired probability that it would be full and spill. Hence, let

$$S = \bar{S} + t\sigma_s \tag{8}$$

Where t is standard measure of probability of a normal distribution, and σ_s is standard deviation of reservoir contents. Since $\sigma_s = \frac{\sigma_d}{k}$ we can also write

$$S = \frac{\bar{X}}{k} - \frac{b}{k} + t\frac{\sigma_d}{k} \tag{9}$$

But the service function can be set up such that minimum probable discharge from the reservoir

$$b = \bar{X} - t\sigma_d \tag{10}$$

Substituting $t\sigma_d$ from Eq. (10) into Eq. (9) yields

$$S = \frac{2}{k}(\bar{X} - b) \tag{11}$$

Substituting $\bar{S}$ from Eq. (7),

$$S = 2\bar{S} \tag{11a}$$

Under the conditions given, there are some pertinent restrictions to be satisfied. (1) No storage would be needed if b is equal to the minimum flow of the stream. In this case, the natural flow of the stream would provide the required supply. (2) Since the entire flow is to be regulated, there is inverse linkage between b and k. For example, if a high value of b is to be set, the value of k must be low. These conditions are met by the following considerations.

The minimum probable regulated discharge, b, can be expressed

$$b = \bar{X} - t\sigma\sqrt{\frac{k}{2+k}}$$

The minimum probable natural inflow, m, is

$$m = \bar{X} - t\sigma$$

Taking equivalent probabilities for minimum inflow and regulated discharge and solving for k we obtain

$$k = \frac{2(\bar{X}-b)^2}{(\bar{X}-m)^2-(\bar{X}-b)^2} \tag{12}$$

Note that the greater the value of b, the lower is the value of k.

We can combine Eqs. (11) and (12)

$$S = \frac{(\bar{X}-m)^2-(\bar{X}-b)^2}{(\bar{X}-b)} \tag{13}$$

We can also write

$$S = t\frac{\sigma^2-\sigma_d^2}{\sigma_d} \tag{14}$$

These are general equations for storage for given values of minimum natural flow, and minimum regulated discharge. Note that storage is zero where b = m, and that storage increases rapidly as b approaches $\bar{X}$, and becomes infinite when b = $\bar{X}$. There is a close analogy with ordinary queues. If attempt is made to equate service capacity to mean arrival rate, length of queue becomes infinitely long. (Churchman et al, p. 398 and p. 401, 1957.)

Although Eqs. (13) and (14) appear quite general in form, in that they contain the pertinent factors generally considered in reservoir operation, the equations cannot be applied indiscriminately. They apply only under the conditions for which they were derived, viz.

1. Random inflow, normally distributed.
2. Service function D = b + kS.
3. Value of b, minimum probable discharge (at which reservoir is empty) has the same probability of occurrence as m, minimum observed unregulated (natural) flow.

Modification for Non-Random Inflows

The preceding analysis is based on random occurrence of inflows to the reservoir. The particular implication of a random model that concerns us is

that there is no relationship between sequential flows. In terms of streamflow, this means that a drought is as likely to follow a high year as another drought. However, it has been observed that there may be a sequential relationship—something in the nature of persistence. Abnormalities tend to persist. Without attempting to prove or disprove the reality of persistence, it is possible to include such effects in the storage analysis.

The first step is to describe the persistence or sequential relationship. A simple expression would be $r_n = r^n$ where r_n is the correlation coefficient between items n apart. A more general expression of the sequential effect would be $r_n = ra^{n-1}$ where r is the coefficient of correlation between successive items (n = 1), ra is the correlation between alternate items (n = 2) and so on. The values of a and r are less than 1.

The standard deviation of the discharges as given by Eq. (6) presumes independence between successive items. If, however, successive items are correlated by r, alternate items by ar, and that between n items by $a^n r$, it is possible to revise the equation to include the effect of the linkage. The revised equation is

$$\sigma_d = \sigma \sqrt{\frac{k}{2+k}} \sqrt{1 + \frac{2r}{1+k-a}} \tag{6a}$$

The expression $\sqrt{1 + \frac{2r}{1+k-a}}$ is therefore a correction factor (always greater than unity) to allow for the effect of serial correlation. Introducing the correction factor, c, into Eqs. (13) and (14) for storage we obtain

$$S = \frac{c^2(\bar{X}-m)^2 - (\bar{X}-b)^2}{X-b} \tag{13a}$$

and

$$S = t\,\frac{c^2\sigma^2 - \sigma_d}{\sigma_d} \tag{14a}$$

It will be observed that the effect of non-random distribution of inflows is to increase storage requirements.

Probability Routing

Example 1 illustrates an application of discharge routing to a queuing problem. This application is a simple one of normal inflow and a linear service function. But wherever the frequency distribution of inflows is non-normal and service functions are non-linear, a more flexible method is needed. These are based on a general solution of the queuing equation that is adaptable to all cases, including that in the preceding example, if one wished to handle it by this general method. The procedure involves a kind of "probability routing" in lieu of the more familiar discharge reservoir routing. The answers are obtained from a solution of the single-step queuing equation by the method of finite differences. The single-step queuing equation seeks answers to the question, what is the probability that there will be n units in the queue at the end of a given time interval? This probability is the sum of the independent probabilities that (1) there were n units in the queue at the end of the previous interval and there were no arrivals or departures during the time interval, (2) that there were n units in the queue at the end of the

previous interval, and that arrivals and departures were equal in number, (3) there were n + 1 items in the queue at the end of the preceding interval but that departures exceeded arrivals by one unit, and (4) there were n - 1 items in the queue at the end of the preceding interval but that arrivals exceeded departures by one unit. Formal and general solutions are possible from this statement, provided several restrictions can be made—the time interval is infinitely small, the probability of more than one unit arriving in a time unit is negligible, and that servicing and arrival rates can be expressed formally and simply. None of these restrictions are applicable in the case of reservoir design. However, none of these restrictions are involved in the basic statement of the queuing equation, which can be generalized so as to apply to a wide array of problems.

Example 2

The procedure is best shown by an example. The service function for the selected example is defined by two lines shown on Fig. 2. The distribution of inflows is shown on Fig. 3. The distinguishing feature of this example is the irregular service function and the non-normal distribution of inflows. The problem is the determination of the distribution of storage and discharge.

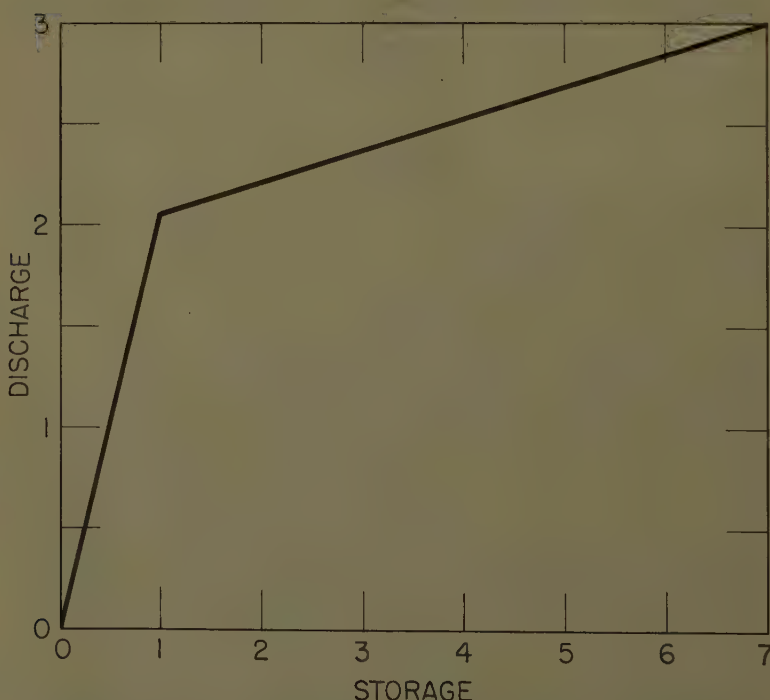

FIGURE 2.--SERVICE FUNCTION, EXAMPLE 2.

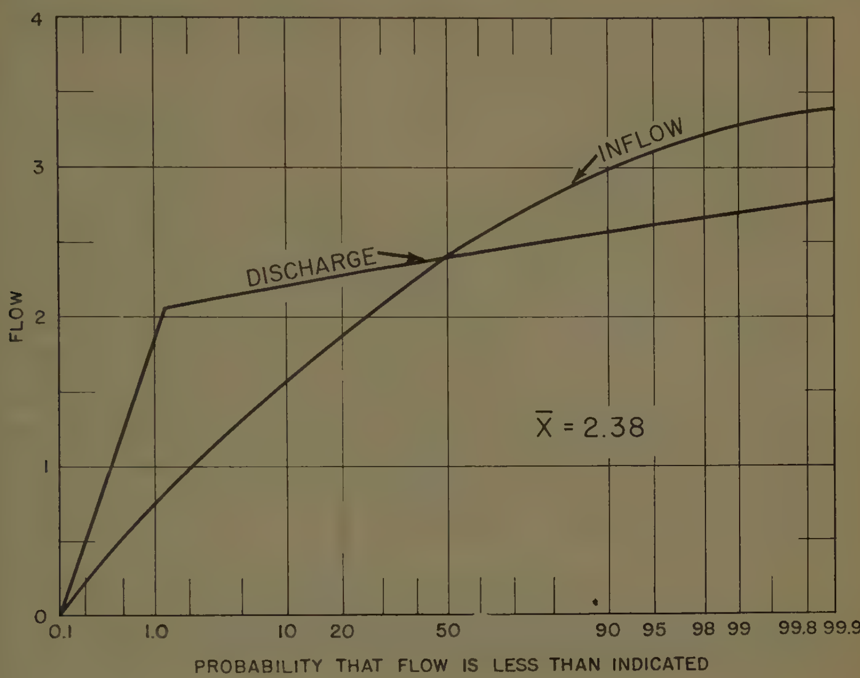

FIGURE 3.--FREQUENCY DISTRIBUTION OF INFLOWS AND DISCHARGES, EXAMPLE 2

I. Probability of storage equal to or less than 1.0 at time t_n.

1. Storage of 1.0 or less can exist at time t_n, if storage at time t_{n-1} was zero and inflow during this interval exceeded discharge by 1.0 or less. Since discharge for a storage of zero is zero (see Fig. 2), and since discharge for a storage of 1.0 is 2.05, average discharge during an interval in which storage increased from 0 to 1 would be 1.02. Hence, inflow during this interval must be 1 plus 1.02 or 2.02 to produce an increment in storage from zero to 1.0. The probability of an inflow of 2.02 or less according to Fig. 3 is 0.27. The partial probability for this combination is 0.27 P, which means that if a storage of zero exists, then there is a 27 per cent chance that the storage will be 1.0 or less at the end of the next interval of time.
2. Storage of 1.0 or less can exist at time t_n, if storage at time t_{n-1} was less than 0.5 but more than zero, and if inflow during this interval exceeded discharge by a sufficient amount. Since discharge for a storage of 0.25 (mid range value) is 0.5, and since discharge for a storage of 1.0 is 2.05, average discharge during an interval in which storage increased from 0.25 to 1.0 would be 1.28. Hence inflow during this interval to produce an increment in storage from 0.25 to 1.0 must be 0.75 + 1.28 or 2.03. The probability of an inflow of 2.03 or less

according to Fig. 3 is 0.27. The partial probability for this combination is 0.27 $P_{0-0.5}$, which means that if storage in the range 0 to 0.5 exists, then there is a 27 per cent chance that the storage will be 1.0 or less at the end of the next interval of time.

3. Similarly computations are carried out for various combinations. For example, the computations for initial storage between 2.5 and 3.0 are in the following paragraph.
4. Storage of 1.0 or less can exist at time t_n if storage at time t_{n-1} was less than 3.0 but more than 2.5 if inflow during this interval was less than discharge by a sufficient amount. Since discharge for a storage of 2.75 (mid-range value) is 2.32, and since discharge for storage of 1.0 is 2.05, average discharge during an interval in which storage decreased from 2.75 to 1.0 would be 2.18. Hence inflow during this interval must be -1.75 + 2.18 = 0.43 or less. The probability of an inflow of 0.43 or less according to Fig. 1 is 0.004. The partial probability for this combination is 0.004 $P_{2.5-3.0}$, which means that if storage is in the range from 2.5 to 3.0, then there is a probability of 0.004 that storage will be 1.0 or less at the end of the next interval of time.
5. The results of this series of computations can be written

$$P_{1.0} = 0.27P_0 + 0.27P_{0-0.5} + 0.27P_{0.5-1.0} + 0.20P_{1-1.5} + 0.06P_{1.5-2.0} + 0.016P_{2-2.5} + 0.004P_{2.5-3.0}$$

in which P_0 is the probability that the storage at the beginning of the period is zero. This equation states that the total probability that storage at the end of any period (therefore at any time) will be 1.0 or less is the sum of the several partial probabilities.

II. The same computations are carried for other total probabilities P_0 to $P_{7.0}$ being carried high enough to assure that all plausible amounts of storage are included. The equations are listed in Table 1. Note that vertically each coefficient increases from zero to unity. If storage is between 3.0 and 3.5, the probability is zero that storage will be 1.0 or less at the end of the next interval. It is virtually certainty (= 1.00)that it will be 4.5 or less at the end of the next interval.

III. The problem is to solve this set of equations for $P_0 \ldots . P_7$. Recognizing that these are cumulative probabilities, we can compute the expressions for storage in the successive ranges by successive subtraction, thus $P_{2-2.5} = P_{2.5} - P_{2.0}$. The results are given in Table 2. Note that coefficients add up to 1.0 vertically. Each horizontal line is an equation: there is an implied equal sign between the first and second columns. (For example, the second line reads $P_{0-0.5} = .02P_{0-0.5} + .02P_{0.5-1.0} + .01P_{1.0-1.5} + .003P_{1.5-2.0}$. Thus we have a set of simultaneous equations to be solved. The simplest method appears to be successive approximation. Any reasonable set of values can be used for the first approximation.

If no other initial estimate can be made, all probabilities can be assumed to be the same. The first approximation values are inserted in each equation, to derive the second approximations. These are reinserted to derive a third approximation, and the process is continued until the difference between inserted and derived figures becomes negligible. Care must be taken in the computations that the sum of the probabilities after each trial equals one. The values in the column labeled "Ans." of Table 2, were

Table 1.--First set of probability equations, example 2

Probability at end of interval	Probability at beginning of interval														
	P_0	$P_{0-0.5}$	$P_{0.5-1}$	$P_{1-1.5}$	$P_{1.5-2}$	$P_{2-2.5}$	$P_{2.5-3}$	$P_{3-3.5}$	$P_{3.5-4}$	$P_{4-4.5}$	$P_{4.5-5}$	$P_{5-5.5}$	$P_{5.5-6}$	$P_{6-6.5}$	$P_{6.5-7}$
P_0	0	0	0	0	0										
$P_{0.5}$	.02	.02	.02	.01	.003	0	0								
$P_{1.0}$	.27	.27	.27	.20	.06	.016	.004	0							
$P_{1.5}$	.90	.85	.40	.34	.20	.07	.02	.005	0						
$P_{2.0}$	1.00	1.00	.97	.88	.45	.25	.08	.02	.01	0					
$P_{2.5}$			1.00	1.00	.90	.62	.30	.10	.02	.008	0				
$P_{3.0}$					1.00	.94	.67	.35	.12	.04	.01	0			
$P_{3.5}$						1.00	.97	.74	.40	.18	.06	.016	0	0	
$P_{4.0}$							1.00	.97	.74	.40	.18	.06	.016	.004	0
$P_{4.5}$								1.00	.99	.80	.48	.20	.07	.02	.006
$P_{5.0}$									1.00	1.00	.85	.50	.24	.08	.02
$P_{5.5}$											1.00	.90	.58	.30	.10
$P_{6.0}$												.997	.94	.66	.35
$P_{6.5}$												1.00	1.00	.97	.72
$P_{7.0}$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 2.--Second set of probability equations, example 2

Probability at end of interval	Probability at beginning of interval																
	P_0	$P_{0-0.5}$	$P_{0.5-1}$	$P_{1-1.5}$	$P_{1.5-2}$	$P_{2-2.5}$	$P_{2.5-3}$	$P_{3-3.5}$	$P_{3.5-4}$	$P_{4-4.5}$	$P_{4.5-5}$	$P_{5-5.5}$	$P_{5.5-6}$	$P_{6-6.5}$	$P_{6.5-7}$	Ans.	$\sum$
P_0	0	0	0	0	0											0	0
$P_{0-0.5}$	.02	.02	.02	.01	.003	0	0	0								0	0
$P_{0.5-1}$	.25	.25	.25	.19	.057	.016	.004	0								.012	.012
$P_{1-1.5}$	.63	.58	.13	.14	.14	.054	.015	.005	0	0						.025	.037
$P_{1.5-2}$	.10	.15	.57	.54	.25	.18	.06	.015	.01	0						.068	.105
$P_{2-2.5}$	0	0	.03	.12	.45	.37	.22	.08	.01	.008	0					.137	.242
$P_{2.5-3}$			0	0	.10	.32	.37	.25	.10	.032	.01	0				.193	.435
$P_{3-3.5}$					0	.06	.30	.39	.28	.14	.05	.016	0			.217	.652
$P_{3.5-4}$						0	.03	.23	.34	.22	.12	.044	.016	.004	0	.156	.808
$P_{4-4.5}$							0	.03	.25	.40	.30	.14	.054	.015	.006	.121	.929
$P_{4.5-5}$								0	.01	.20	.37	.30	.17	.06	.014	.054	.983
$P_{5-5.5}$									0	0	.15	.40	.34	.22	.08	.015	.998
$P_{5.5-6}$											0	.097	.36	.36	.25	.002	1.00
$P_{6-6.5}$												.003	.06	.31	.37	0	
$P_{6.5-7}$												0	0	.03	.28		

obtained after a third approximation using a slide rule for the multiplications. Values of zero in this column must be recognized as nominal values. The last column of Table 2 gives the cumulative probabilities in the values of P_0, $P_{0.5}$, etc.

IV. Fig. 4 shows a plot of the calculated frequency distribution of storage as defined by the results in the last column of Table 2. This gives the answer sought. The frequency distribution of discharge from the reservoir can be readily determined from Fig. 4 by use of the service function on Fig. 2. For example, storage is equal to or less than 2.7 for 30 per cent of the time. The corresponding discharge according to Fig. 2 is 2.3. The frequency distribution of discharges on Fig. 3 was built in this way.

Example 3

In this example, a reservoir of capacity 2 is to be operated so that the discharge rate does not exceed 1.15. The service function is therefore of Type IIB. The problem is to define the frequency distribution of storage and discharges.

The procedure is exactly as before, except there are upper and lower bounds to the storage. The probability equations are given in Table 3. The frequency distribution of storage is defined on Fig. 5. The distribution of discharges, also shown on Fig. 5, requires some further explanation. The discharges equal 1.15 while there is storage and the reservoir does not spill—in this example the probability of spill is very small. The probability of

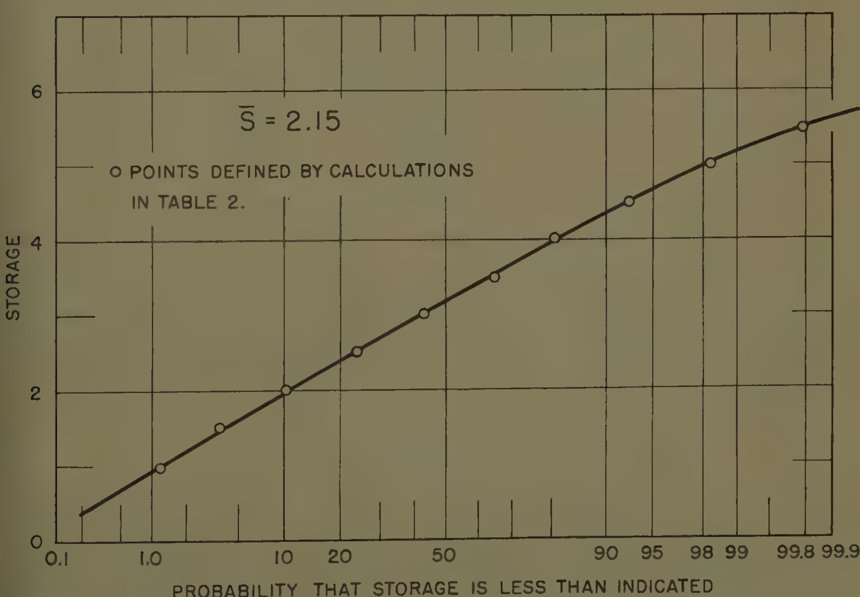

Figure 4.--Calculated frequency distribution of storage, example 2.

Table 3.--Probability equations for example 3

Probability of storage at end of interval	Probability of storage at beginning of interval											
	P_2	$P_{2-1.75}$	$P_{1.75-1.5}$	$P_{1.5-1.25}$	$P_{1.25-1.0}$	$P_{1.0-.75}$	$P_{.75-.50}$	$P_{.50-.25}$	$P_{.25-0}$	P_0		
P_2	0.71	0.81	0.92	0.97	0.989	0.996	0.998	0.999	1.00	1.00		
$P_{1.75}$	.49	.60	.81	.92	.97	.989	.996	.998	.999	1.00		
$P_{1.50}$	.22	.36	.60	.81	.92	.97	.989	.996	.998	.999		
$P_{1.25}$	.06	.13	.36	.60	.81	.92	.97	.989	.996	.997		
$P_{1.00}$	.005	.03	.13	.36	.60	.81	.92	.97	.988	.993		
$P_{.75}$	0	0	.03	.13	.36	.60	.81	.92	.97	.98		
$P_{.50}$			0	.03	.13	.36	.60	.81	.92	.95		
$P_{.25}$				0	.03	.13	.36	.60	.81	.87		
P_0					0	.03	.13	.36	.60	.71		
											Ans.	Σ
$P_{>2}$	.29	.19	.08	.03	.011	.004	.002	.001	0	0	0	- 1.00
$P_{2-1.75}$	.22	.21	.11	.05	.019	.007	.002	.001	.001	0	.001	.999
$P_{1.75-1.5}$	.27	.24	.21	.11	.05	.019	.007	.002	.001	.001	.003	.996
$P_{1.5-1.25}$	.16	.23	.24	.21	.11	.05	.019	.007	.002	.002	.006	.990
$P_{1.25-1.0}$	.055	.10	.23	.24	.21	.11	.05	.019	.008	.004	.012	.978
$P_{1.0-.75}$	.005	.03	.10	.23	.24	.21	.11	.05	.018	.013	.029	.949
$P_{.75-.50}$	0	0	.03	.10	.23	.24	.21	.11	.05	.03	.057	.892
$P_{.50-.25}$			0	.03	.10	.23	.24	.21	.11	.08	.111	.781
$P_{.25-0}$				0	.03	.10	.23	.24	.21	.16	.183	.598
P_0					0	.03	.13	.36	.60	.71	.598	0

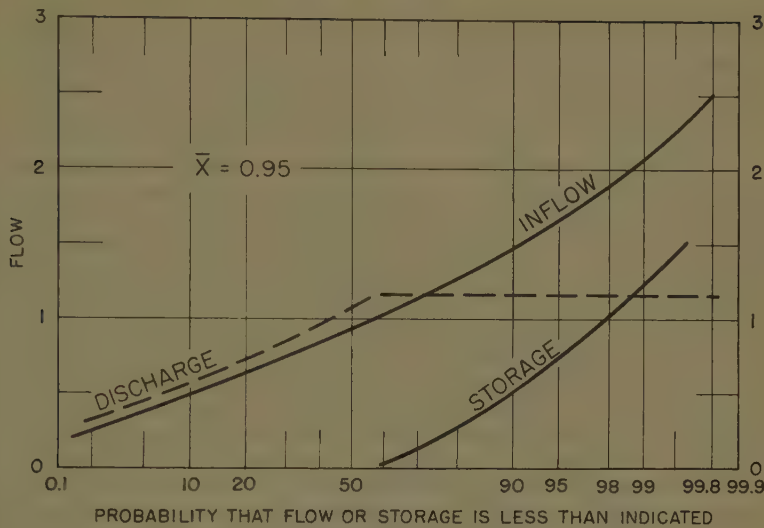

Figure 5.--Frequency distributions of inflow, storage, and discharge, example 3.

discharges less than 1.15 are computed as follows: The probability of a discharge of 0.5 or less is the sum of the partial independent probabilities that there is no storage and inflow is 0.5 or less; that storage is in the range 0 - 0.25 and inflows are 0.37 or less; that storage is in the range 0.25 - 0.50 and inflows are 0.12 or less. The calculations are given below:

Storage	Prob.[a]	Inflow	Prob.	Partial prob.
0	0.598	0.5	0.11	0.066
0 - 0.25	.183	.37	.05	.009
0.25 - .50	.111	.11	.005	0
				.075

a. From Table 3.

The probability of a discharge of 0.5 or less is therefore 0.075 which establishes one point on the discharge frequency curve. Other points are computed to define the graph shown on Fig. 5. As a final check the average discharge must equal the average inflow.

Example 4
(Storage Analysis Considering Monthly Regimen)

The analysis previously described is based on the simple case of uniform flow and draft within each unit of time. However, if the time unit is long—say, a year—the condition may no longer be met. The general solution might be to develop the queuing equations for each month. In the case of years, the probability distribution of storage at the beginning of years is the same as at the ends. In the case of months, this is no longer true, as there is a seasonal

progression in the probability distribution of storage. The matter can be solved by considering the probability distribution for the end of one calendar month is the same as the probability distribution of storage for the beginning of the next following calendar month. Thus, 12 separate sets of queuing equations are set up, to be solved as a single system of equations. Tables can be set up for each month similar to Tables 1 and 2, except that the solution now involves a simultaneous solution of all 12 months. The solution tends to be long, hence a simpler method permitting use of the year as the unit of time for queuing would be useful.

The record of Cedar River near Landsburg, Wash., from 1896-1950 is used to illustrate a procedure. Fig. 6 shows the duration curve of annual flows. Cedar River, like streams generally in the Pacific Northwest, has a highly seasonal regimen; flow reaches a maximum in the winter and a low in the late summer. Fig. 7 shows the monthly median discharges. To emphasize the seasonal contrasts, demands, as shown in Fig. 7, are assumed to be minimum in winter and maximum in summer, such as might be expected by combined water supply and irrigation requirements. Thus, a reservoir would have a twofold job—to store water in winter for summer use, and to store water in "wet" years to supplement the flow of "dry" years. A reservoir of 250,000 acre-feet capacity is assumed. The problem is to determine the frequency distribution of storage, probability of spilling, and probability of being empty for each month of the year.

The principal modification is in the basic equation: inflow (I) equals required outflow (D) plus change in storage (ΔS). This equation cannot be applied to long units of time during which there is apt to be considerable variation in

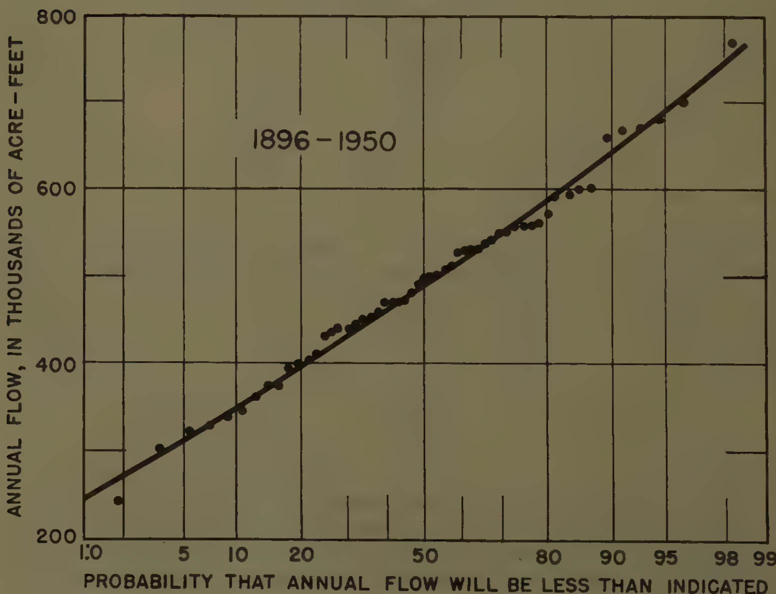

Figure 6.--Duration curve of annual flows Cedar River.

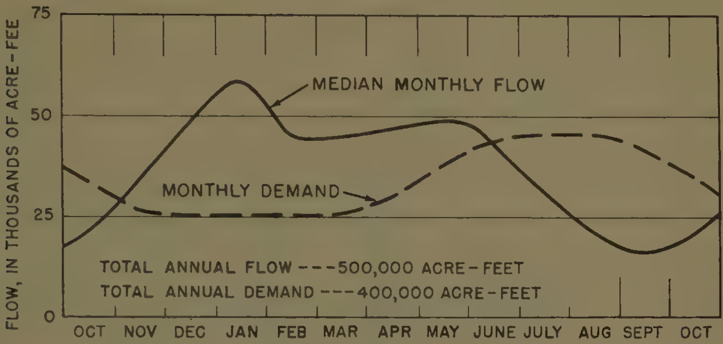

Figure 7.--Median monthly flow and monthly demand, Cedar River example.

inflow and demand, with the likelihood of spillage or an empty reservoir during the interval. Excess flow during spillage is not available to offset subsequent deficiencies in flow nor can deficiencies while the reservoir is empty be compensated for by subsequent excesses in flow. The procedure therefore is to prepare graphs such as on Figs. 8 and 9 that for Cedar River show the net annual change in storage in relation to the annual inflow and the initial storage. These graphs are prepared by first determining the average distribution of the monthly flows during years with various amounts of total flow. These monthly flows are then routed through a reservoir of 250,000 acre-feet capacity to determine the net annual change in storage. The routing is carried out for twelve-month periods ending with each calendar month. Fig. 8 applies to the year ending October 31, in the low-water period, and Fig. 9 to the year ending May 31 in the high-water period of the year. These diagrams are used in lieu of the equation $I = D + \Delta S$ to determine annual inflows required to produce a specified net annual change with a given initial storage. The subsequent procedure is exactly the same as previously described, leading to a frequency distribution of storage for each calendar month. However, we have not yet considered the effect of sequential correlation, so we therefore consider these frequency distributions as "trial solutions".

Sequential Correlation

The preceding analysis is based on the condition that inflows are random in occurrence—that the probability of an inflow as given by the duration curve is independent of preceding flows. Without in any way attempting to prove or disprove the existence of non-random tendencies in stream flow (other than that due to carry over between days and months), probability routing procedure can be applied to the determination of storage even where streamflow data are correlated in time sequence.

When inflows are random the probabilities of inflows are independent of anticedent discharges or of the amounts of storage on hand. Where inflows are related, then it is necessary to recognize that the probability of inflows depend on what went on before. If a reservoir is near empty as a result of a

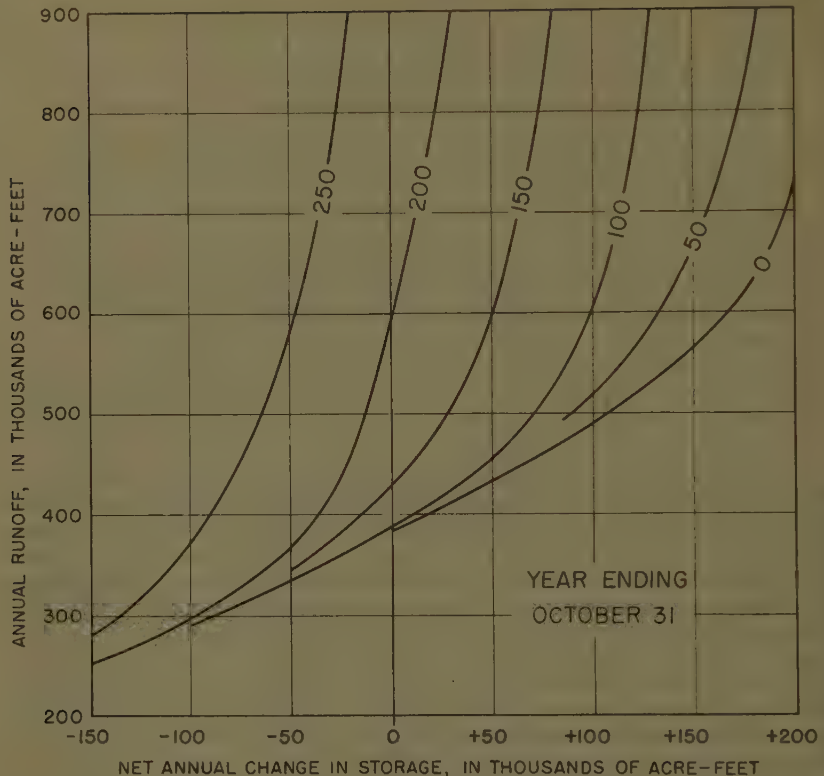

Figure 8.--Net change in storage in relation to initial contents, year ending Oct. 31.

period of low inflow, then the probability of high inflow would be less than if the reservoir is full. As was observed before, the effect of serial correlation is to increase the range in storage.

The problem therefore is to obtain a clue as to the proper frequency distribution in the probability routing. It is necessary to define duration curves that are conditional on certain flows having occurred. If the coefficient of correlation, r, between successive items is computed, then the mean and standard deviation of the duration curves can be determined as follows:

$$Xm = \overline{X}(1-r) + rX \quad \text{and}$$

$$\sigma' = \sigma\sqrt{1-r^2}$$

When Xm is the mean of the duration curve of the flows that follow in the time interval immediately after a flow X, σ' is the standard deviation of the prospective flows, $\overline{X}$ is the general mean, and σ is the standard deviation of the whole array of flows. Since r is generally small, say 0.3 or less, σ' equals σ very nearly.

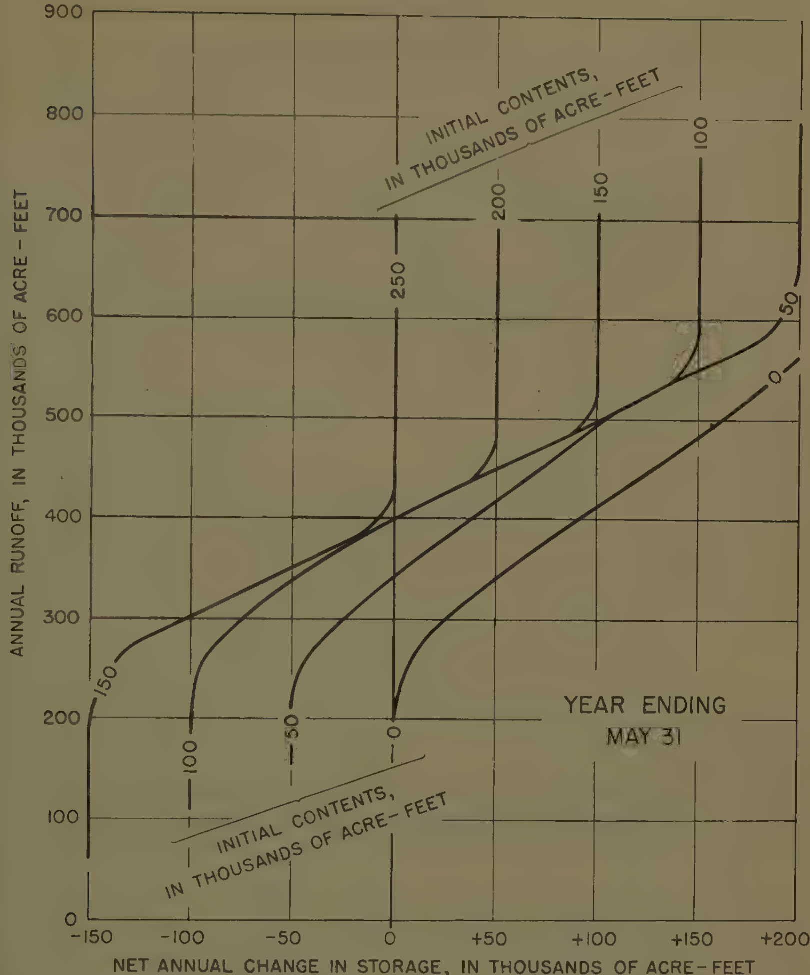

Figure 9.--Net annual change in storage in relation to initial contents, year ending May 31.

Another procedure which was followed in the Cedar River example was to divide the array of annual discharges into three groups, each containing an equal number of years: viz. an upper third (those above 540,000 acre-feet), a middle third (those between 540,000 and 450,000), and a lower third (those below 450,000 acre-feet). Separate arrays were then prepared of the annual discharges in the years following those in the upper third, middle third, and lower third. The resulting frequency distributions are shown on Fig. 11. If there were no sequential correlation, the three frequency distributions would nearly coincide, but as it is, there is a noticeable difference. The frequency distribution of the annual flows following those in the upper third is above the others, and vice versa for the frequency distribution of the flows that follow those—the lowest third. The dispersion is equivalent to a sequential correlation coefficient of 0.28.

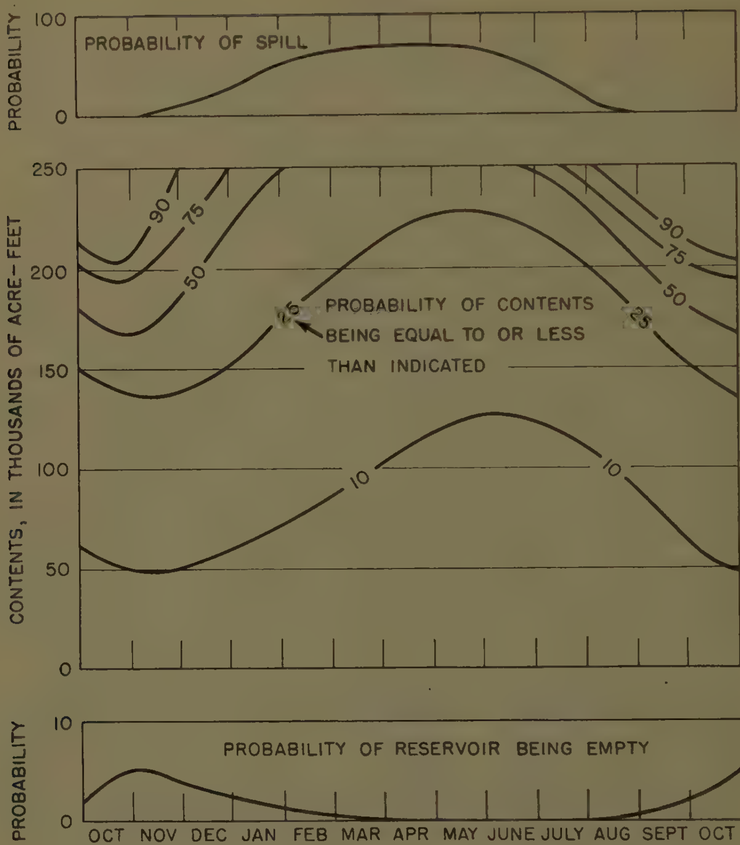

Figure 10.--Results of storage computations Cedar River, Wash.

Having completed trial solutions previously mentioned using the single duration curve as given in Fig. 6, the probability routing is now repeated using the conditional duration curves shown on Fig. 11, as follows:

The trial solutions of the frequency distribution of reservoir contents are used as guides to the selection of the proper conditional duration curves (Fig. 11) to be used in the probability routing, based on the fact that low, medium, or high reservoir contents are associated with low, medium, or high antecedent inflows. Because in this example the conditional duration curves on Fig. 11 were defined by dividing the range of antecedent discharges into equal thirds, the range in storage contents is similarly divided. For example, the trial solution for years ending on October 31 gave results as follows:

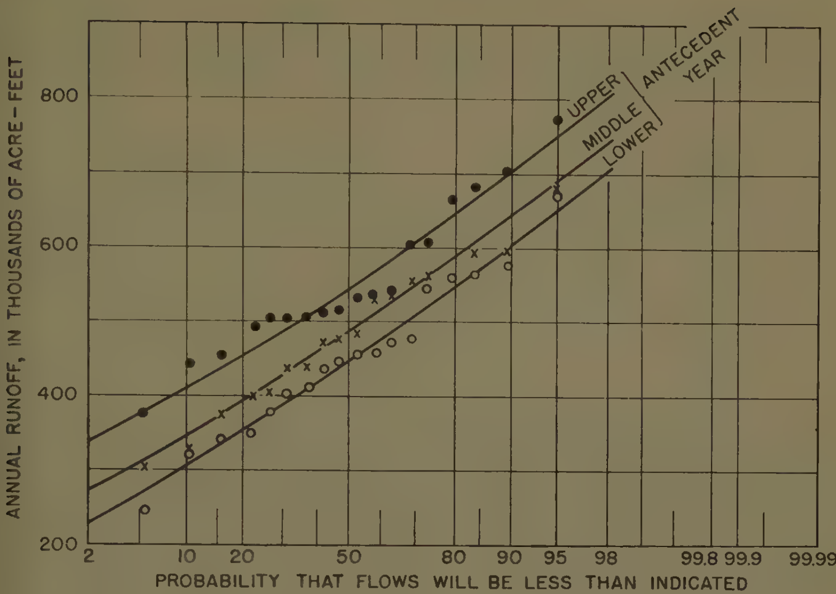

Figure 11.--Variation of annual flow of Cedar River depending on antecedent flow.

Contents (1,000 acre-feet)	Probability of contents less than indicated
0	0.014
50	.044
100	.105
150	.261
200	.807
250	1.000

A graph of the above data indicates that on October 31 contents are less than 160,000 acre-feet for one-third of the time, and are less than 190,000 acre-feet for two-thirds of the time. Therefore, in the probability routing for October 31, for initial contents less than 160,000 acre-feet, the lower duration curve is used, for initial contents between 150,000 and 190,000 acre-feet, the middle curve is used, and for initial contents above 190,000 acre-feet the upper curve is used.

For May 31 the trial solution indicates that contents are less than 250,000 acre-feet for one-third of the time, and the reservoir is spilling for two-thirds of the time. Thus the lower duration curve is used in the probability routing where initial contents are less than 250,000 acre-feet, and a duration curve intermediate between the middle and upper curves when initial contents are 250,000 acre-feet.

These computations lead to a second solution of the frequency distribution of reservoir contents. If this differs greatly from the first or trial solution, a third solution may be indicated. In this example this was not done. A comparison of the results of the first and second solutions for October 31 is as follows:

Contents (1,000 acre-feet)	Probability of contents less than indicated	
	First solution	Second solution
0	0.014	0.045
50	.044	.085
100	.105	.158
150	.261	.309
200	.807	.795
250	1.000	1.00

The results of the second solution have been graphed on Fig. 10. The upper graph shows the probabilities of the reservoir spilling during the several months of the year. The probability of spilling reaches about 70 per cent during March, April, or May and is virtually zero during September or October. The central graph shows the probability of reservoir contents of indicated amounts during the course of the year. For example, reservoir contents in 25 per cent of Novembers will be less than about 135,000 acre-feet. Reservoir contents in about 25 per cent of the Mays will be less than about 230,000 acre-feet. The lower graph shows the probability of the reservoir being empty during the several months of the year. The probability of an empty reservoir is virtually zero from April through July, and is about 5 per cent during October. A decision whether a 250,000 acre-foot capacity reservoir is satisfactory or not depends on the economic consequences of spilling or of the reservoir being empty. Spillage may be the cause of flood damages. While the reservoir is empty, discharges will be less than needed.

The difference between the first or trial solution and the second solution is in the direction of increasing the frequency of spilling during the wet season and increasing the frequency of an empty reservoir in the dry season, in other words adding to storage requirements. But this effect can be offset by using the correlation tendency as a forecast tool in adjusting reservoir releases. In a sense this is already done in practice whenever reservoir releases are curtailed during a period of dwindling reserves and increased as reservoir contents approach the capacity of the reservoir.

Evaporation

Evaporation draft is a Type III service function (see Fig. 1). It therefore can be handled by combining the intended operational service function with a Type III function. If the operational service function is, say, Type I, then the combined service function is a series of graphs showing relation between storage and discharge varying seasonally.

If the operational service function is Type II, the combined service function is a family of lines varying seasonally. The frequency of storage can be computed directly.

The frequency of discharge, computed as previously explained, will yield total discharge plus evaporation. As the losses due to evaporation are

included in the service function they must be subtracted from the computed discharges. Given the season and probability of various amounts of storage, the computed discharges can be corrected for the evaporation loss.

Discussion

The method described was prompted by considerations of the operations of queues. Queuing theory is used as the model for synthesis of the essential characteristics of streamflow (such as the frequency distribution and the sequential correlation) in the solution of reservoir storage problems. Analyses and synthesis according to the queuing theory offers some advantages. The procedure seems to avoid some of the difficulties of storage analyses made by use of the traditional mass curve or by trial routing of historic discharge through reservoirs. The results of such computations are biased by the length of record available. The longer the record, the greater the storage that will be indicated—for example, Hurst (1950) showed that storage needed to average out a fluctuating discharge increases as the 0.72 power of the length of record. The system of "probability routing" described in this paper also avoids the vexing question of what initial storage is to be used in the customary discharge routing or mass-curve study. Probability routing, moreover, is an exact procedure, sometimes called "nonparametric" by which is meant that the combination of probabilities is unaffected by the kind of frequency distribution back of the probabilities.

Sequential correlation is a problem not only in this application of queuing theory, but in all methods of storage analyses. The problem is more evident in queuing theory because sequential correlation must be clearly expressed. Sequential correlation is obscured in storage analyses by the mass curve because the available record is taken "as is". Where storage planned is but a small fraction of the annual flow, such tacit expression of sequential correlation may be satisfactory. However, for large and important storage developments, the simplicity of this treatment may be misleading and may represent inefficient use of the available record which contains only one of the many possible combinations that may exist among the river flows. It appears more efficient to analyze the record to discern the underlying factors in its composition, of which the sequential relation is one, and to use this information in a model upon which to erect all possible combinations, rather than to limit oneself to the single combination offered by the limited record.

The probability routing automatically assures that all possible combinations of inflows and discharges are reflected in the results. In this way it is tantamount to results that would be obtained from an infinite array, subject only to the sampling error of the duration curve and the desired refinement in selecting limits for the computations. It surmounts the difficult selection of a "critical" period for storage design. Reservoir design on the basis of a selected "critical" low water period is likely to be quite variable from one stream to another. The probability method permits establishment of design criteria on a basis of uniform risks.

The utility of obtaining results of a storage analysis directly in terms of probability becomes most evident when operational decisions must be made. It will be remembered that the probability curves on Fig. 10 are the resultants of all possible combinations of initial storages and prospective flows. In actual operations, contents at a given time are known and thus the probabilities of

future contents can be calculated starting with the known storage as a base. If forecasts of prospective flows are available, then these can be incorporated, provided they are coupled with a statement of the standard error, to enable one to construct the probability distribution of the prospective flows. This forecast distribution is then used in lieu of the distribution defined by the whole array of past flows. The results of this analysis leads, for example, to more definite estimates of the probabilities of the reservoir spilling or being empty. These estimates in turn may dictate operational revisions of the service function in order to minimize undesirable risks.

Once the procedure is understood, the computations are fairly rapid—they take longer to describe than to carry out. It is only essential to remember when to multiply and when to add probabilities. The basic rules are that the probabilities of contingent events are multiplied, those of alternate events are added.

REFERENCES

1. Churchman, C. W., Ackoff, R. L., Arnoff, E. L., Introduction to Operations Research, John Wiley & Sons, New York, 1957.
2. Hurst, H. E., Long-Term Storage Capacity of Reservoirs, Trans. Am. Soc. Civil Engrs., v. 116, pp. 770-808, 1951.
3. Moran, P. A. P., A Probability Theory of Dams and Storage Systems, Australian Jour. of Applied Sci., v. s., pp. 116-124, 1954.
4. Rippl, W., The Capacity of Storage Reservoirs for Water Supply, Proc. Inst. of Civil Engrs., v. 71, pp. 270-278, 1883.

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CONTENTS

DISCUSSION
(Proc. Paper 1832)

(Over)

MECHANICS OF SEDIMENT-RIPPLE FORMATION[a]

Discussion by John L. Bogardi
Closure by Hsin-Kuan Liu

JOHN L. BOGARDI.[1]—The paper by Mr. Liu should be considered pioneering in all respects. The only general remark that can be made in connection with Mr. Liu's paper is that the reader cannot help but perceive a slight feeling of incompleteness in noting the mere reference to the study of Kalinske, without, however, pointing out directly that parameters U_*/ω and U_*d/γ are identical to $1/t$ and $\underline{P}$ respectively although the latter were used over ten years ago by Lane and Kalinske. It should be noted at this juncture that as is commonly known a number of attempts have been made during the historical development of the theory of sediment transportation to introduce dimensionless parameters. To quote a single example: the parameter $\frac{\rho_s - \rho_f}{\rho_f} \frac{d}{SD}$ introduced by Einstein(1) and referred to as flow intensity is essentially identical with the channel stability factor d/DS in use since 1942 in Hungary(2) and is an invariant proportionate to the square of parameter $\underline{t}$. A correlation is indicated—and certainly would not have presented difficulties to Mr. Liu—of the multitude of different parameters. Also, a definition is needed which includes the interrelation and the application of the parameters to sediment movement as clearly as our present theoretical understanding of the problem will permit.

Inspection of Fig. 10 reveals that along the transposed Shields' curve within the validity of Stoke's law, where $w \sim d^2$

$$\frac{U_*}{w} = C_1 \left(\frac{U_* d}{\nu}\right)^N \tag{1}$$

in which N is the approximate slope of the curve showing the variation of U_*/ω with U_*d/γ whence

$$U_*^{\,1-N} = C_2\, d^{\,2+N} \tag{2}$$

For N = -2

$$U_* = C_3 d^{\circ} = \text{Constant} \tag{3}$$

a. Proc. Paper 1197, April, 1957, by Hsin-Kuan Liu.
1. Prof. of Hydraulic Eng., Univ. of Budapest, Hungary.

which means that the movement of very small particles is not affected by the shear velocity—that is, by the magnitude of the shear. The correctness of this assumption is obvious if it is remembered that neither shear velocity nor shear have an affect upon the movement of small particles.

The fall velocity within the intermediate range from d = 0.1 to 2 mm varies according to different powers of particle size. Assuming again $\omega \propto d^{5/4}$

$$U_*^{1-N} = C_4 d^{(5/4+N)} \tag{4}$$

if N is chosen as -1/2 as an average

$$U_* = C_5 d^{1/2} \tag{5}$$

For large particles

$$\omega \propto d^{1/2}$$
$$U_*^{1-N} = C_6 d^{\frac{1}{2}+N} \tag{6}$$

Since for this range N = 0

$$U_* = C_7 d^{\frac{1}{2}} \tag{7}$$

Therefore, for large particles, and even for fractions within the intermediate range, shear velocity is proportionate to the square root of particle size.

Equation 7 is in full agreement with the frictional drag theory of sediment movement since, as it is known that

$$T_c \approx U_*^2 = C_8 d \tag{8}$$

where T_c is the critical tractive force.

REFERENCES

1. Einstein, H. A. "Formulas for the Transportation of Bed Load" Trans. ASCE, Vol. 107, 1942, p. 567.
2. Bogardi, J. "Enquiries into Channel Stability and Sediment Movement Research Institute of Water Economy, Budapest, 1957.

HSIN-KUAN LIU,[1] A.M. ASCE.—The writer welcomes all the preceding discussions. In view of the fact that various comments made by the discussers are related to two major subjects: (a) the relation between the ripple-formation and the transition of the flow near the boundary from laminar to turbulent and (b) the comparison between the Shields' criterion of the beginning of motion and the writer's criterion of the beginning of ripple-formation, these two subjects will be discussed first. Other comments will be discussed according to the order by which they appeared in the journal.

As can be seen from Fig. 9, the ripples form approximately in the transition stage, that is, the shear-velocity Reynolds number is greater than 3 and less than 70. Within this range the transition of the boundary from hydraulically smooth to hydraulically rough can be understood clearly, if the shear-velocity Reynolds number U_*d/γ is interpretated as the ratio of the thickness of the laminar sublayer to the size of the sediment. If the thickness of the laminar sublayer δ' is assumed to be equal to 11.6 γ/U_*,(11) at $U_*d/\gamma = 3$, the laminar sublayer is about 4 times the height of the sediment; therefore the sediment is well covered by the laminar sublayer and the boundary is hydraulically smooth. At $U_*d/\gamma = 70$, the size of the sediment is about 6 times the thickness of the laminar sublayer, the boundary is hydraulically rough, and the flow near the boundary is turbulent. Although this concept is very useful in explaining the hydraulic condition of a plane boundary, the shear-velocity Reynolds number loses this physical significance once the sand waves such as ripples and dunes appear on the alluvial boundary. Therefore it would be better if the shear-velocity Reynolds number is interpreted as an index of instability from the point of view of sediment-ripple formation. According to Dryden,(18) as the boundary changes from smooth to rough, the laminar sublayer becomes wavy first. This leads to the writer's hypothesis that as the laminar sublayer becomes wavy and eventually disintegrates into small eddies, at the same time ripples appear on the alluvial bed. It is further hypothesized therefore that both the turbulence originated from the boundary and the ripples are all caused by the instability of the laminar sublayer. Furthermore, the laminar sublayer is considered as an intermediate zone having a high velocity gradient between the main flow and the alluvial bed. The instability is directly related to (a) the velocity gradient represented by the shear-velocity, (b) the state of the disturbance represented by the sediment size d, and (c) the degree of damping due to viscosity represented by the kinematic viscosity γ. Therefore U_*d/γ is important in classifying the bed configuration.

The writer has proposed that ripples can form under three conditions (a) at the transition of the boundary layer from laminar to turbulent flow, (b) at the transition of the boundary from hydraulically smooth to hydraulically rough, and (c) the instability of vortex sheet at the interface in case the flow near the boundary is already turbulent.

Fig. 9 indicates that laboratory data collected by the writer are mostly related to condition (b). However, a re-examination of the writer's experimental data Runs 1-1 to 1-3 shows that these three runs may be related to condition (a). To illustrate this point, data of Run 1-3 and Fig. 8 will be used.

According to Schlichting's neutral instability curve,(19) the critical length-Reynolds number $U_\alpha x/\gamma$ is 1.12x 10^5 at the point of instability for flow along

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a flat plate at zero incidence, where the flat plate has a very smooth surface, and the approaching flow has a low degree of turbulence. The corresponding wave-length is $\lambda = 22.4\ \delta^*$, where δ^* is the thickness of the laminar boundary layer at x, and is equal to $1.73 \sqrt{x\gamma/U_\alpha}$, where x is the distance from the leading edge, and U_α is the velocity at the edge of the laminar boundary layer. The critical length-Reynolds number for flow in an open channel having an alluvial boundary may be smaller than 1.12×10^5. The effect of boundary roughness on the instability which is not fully known is neglected at present. Ripples shown on Fig. 8 were about at a distance of 8 inches from the entrance. If the corresponding velocity U_∞ is taken as the mean velocity of the flow which is estimated to be 0.78 fps, at the place where the first ripple was found $\frac{U_\alpha x}{\nu} = \frac{0.78 \times 2/3}{1.2 \times 10^{-5}} = 4{,}300$ which is in the same order of magnitude as Schlichting's value. The thickness of the corresponding laminar boundary layer is $\delta^* = 1.73 \sqrt{\frac{\nu x}{U_\alpha}} = 5.54 \times 10^{-3}$ ft the corresponding wave-length is $\lambda = 22.4\ \delta^* = 0.124$ ft. = 1.5 in. As shown in Fig. 8, the measured wave-length is about 2 inches.

If the effect of roughness on the transition from laminar to turbulent flow is considered, according to Schlichting (with reference to some Japanese data) the critical shear-velocity Reynolds number at the stage of instability is $U_* k/\gamma = 15$ where k is the height of roughness. Data of Fig. 8 listed as Run 1-3 indicate that ripples begin at $U_* d/\gamma = 3$ where d is considered to be equal to k. The calculated shear Reynolds number although smaller than 15, is of the same order of magnitude. On the other hand $U_* d/\gamma = 3$ also means the beginning of the boundary transition from hydraulically smooth to hydraulically rough. Hence there is a question as to whether ripples of Fig. 8 are caused under condition (a) or condition (b). The writer thinks that this question cannot be settled till the theory of instability for open channel flow having an alluvial boundary becomes fully known. Nevertheless it is rather clear from this discussion and also Fig. 9, that ripples are associated closely with the transition from laminar flow to turbulent near the boundary.

It is interesting to know that Smoltczyk[37] of Germany has also demonstrated in 1955 that the Schlichting's instability concept can be used in the prediction of ripple length caused by the currents. He found that the wave lengths computed as an average of the upper and lower branch of Schlichting's curve was of the same order of magnitude as the ripple length obtained by him in an experiment. The approach of using the instability of laminar boundary layer to demonstrate the interfacial wave has gained popularity in recent years, for example Wuest[36] of Germany related Schlichting's theory of instability to the generation of surface waves caused by wind.

The third case that the interfacial wave is caused by the vortex sheet phenomenon cannot be demonstrated as yet by laboratory data, because ordinarily when the Reynolds number is high there are some other factors such as surface waves, which can also cause sand waves.

The comparison of the beginning of bed-load movement, and the beginning of ripple formation raises some very interesting questions. For example, Drs. Vanoni and Brooks propose that the ripple-curve should be moved downward to coincide with the Shields' curve; Messrs. Albertson, Simons, and Richardson propose that the ripple-curve should be modified so that it has the

same trend as that of the Shields' curve. The writer feels that both the Shields' curve and the writer's curve are obtained from experimental data. Therefore they can be modified if there is new evidence from laboratory data. Since the proposed changes are not based upon experimental evidence, the writer thinks that such changes are not justified at the present time.

Most observers have obtained, from a statistical point of view, the distinction between the beginning of motion and the beginning of ripples; therefore these two curves should not coincide, when U_*d/γ is less than about 90. Furthermore if it is assumed that the ripples will form as soon as the movement begins, and since the beginning of motion is very difficult to be defined, especially for mixture, the writer would propose for practical purpose that the curve of beginning of ripples should be used in the study of sediment transport rather than that of the beginning motion. Although there is certain relationship between the beginning of motion and the beginning of ripples, it is not necessary that the curve for the beginning of ripples should have the same trend as the beginning of motion as suggested by Messrs. Albertson, Simons, and Richardson.

Before comparing the parameters of Shields' and those of the writer's, the methods of dimensional analysis will be used to demonstrate all the important variables pertinent to this phenomenon. The beginning of movement and the beginning of ripples depend upon the forces of the flow on the sediment particle and upon the resistance of the particle to the flow. The variables for the flow are: D - depth of flow, S - slope of the flow, T_o - boundary shear, u_y - local velocity, g - gravitational constant, μ - dynamic viscosity, ρ - density of the flow, and k - roughness height of the boundary. Since the slope of the flow is defined if the depth of flow and the boundary shear are given, slope can be omitted. Since the velocity distribution u_y is a function of T_o, ρ, μ, k, it can be omitted also. The roughness height in this case is represented by the sediment size d. The variables of the sediment are: d, sediment size, shape factor, and the $\Delta\gamma_s$, the difference of specific gravity between water and the sediment. For given sediment size, given specific gravity, the shape factor can be evaluated if the fall-velocity of the sediment w is known; therefore, the shape factor can be represented by the fall-velocity. In summary we have the following variables pertaining to both the flow and the sediment. D, T_o, g, ρ, μ, d, w, $\Delta\gamma_s$. Through the use of π theorem, we have the following dimensionless term if T_o, d and ρ are used as the repeating variables:

$$\phi\left(\frac{d}{D}, \frac{\rho g d}{T_o}, \frac{\sqrt{\frac{T_o}{\rho}}\,d}{\nu}, \frac{\sqrt{\frac{T_o}{\rho}}}{w}, \frac{T_o}{\Delta\gamma_s d}\right) = 0 \qquad (1)$$

In which d/D is called the relative roughness; $\frac{\rho g d}{T_o}$, is called channel stability factor by Dr. Bogard;(43) $\sqrt{T_o/\rho}\, d/\gamma$, the shear-velocity Reynolds number, $\sqrt{\frac{T_o}{\rho}}\Big/ w$, the movability number; $T_o/\Delta\gamma\, s^d$, an inverse of the flow intensity. Notice that the inverse of the flow intensity for the beginning of motion is called the coefficient of critical tractive force by Shields. If $T_o/\Delta\gamma\, s^d$ is used for water flowing over sand bed, it is proportional to the inverse of the channel stability factor d/R_bS (= gd/U_*^2), where R_b is the hydraulic radius of the sand bed.

The first two terms can be neglected if we re-examine the variables for the flow. It is generally known that the total depth of the flow does not effect the velocity distribution near the boundary according to Karman-Prandtl's theory. If the effect of secondary circulation is neglected, the ratio of the flow-depth to the width of the channel can be neglected; therefore the depth D can be omitted in the consideration. The factor g can be omitted if the surface wave of the flow is not important which is generally true for the case of beginning motion and beginning of ripples. If we omit the D and g as the variables, we have the following dimensionless parameters

$$\phi\left(\frac{U_* d}{\nu}, \frac{U_*}{w}, \frac{T_o}{\Delta\gamma_s d}\right) = 0 \tag{2}$$

where $U_* = \sqrt{T_o/\rho}$.

Equation 2 is the general equation for the beginning of motion or the beginning of ripples. It should be noticed that both D and g may affect the velocity distribution near the bottom, if the surface waves are present in a flow of shallow depth. Under this condition, Eq. (1) should be used. If the sediment is assumed to be spherical, Eq. 2 can be further simplified. For given sediment size and specific gravity at a given temperature, the fall-velocity is determined; therefore only one of the two: the fall velocity w or the difference of specific weight $\Delta\gamma_s$ is necessary. If w is used, we have the following dimensionless parameters for the beginning of ripples or the beginning of motion,

$$\phi\left(\frac{U_* d}{\nu}, \frac{U_*}{w}\right) = 0 \tag{3}$$

which was used by the writer. If the $\Delta\gamma_s$ is used, we have the following equation

$$\phi\left(\frac{U_* d}{\nu}, \frac{T_o}{\Delta\gamma_s d}\right) = 0 \tag{4}$$

which was used by Shields. It can be shown that U_*/w can be computed if the shear-velocity Reynolds number U_*d/γ and $T_o/\Delta\gamma\, s^d$ are given, or $T/\Delta\gamma\, s^d$ can be computed if the shear-velocity Reynolds number and U_*/w are given. The following was suggested to the writer by Dr. Iwagaki

$$w = \sqrt{\frac{4}{3}\,\frac{\Delta\gamma_s}{\rho}\,\frac{d}{C_D}} \tag{5}$$

$$\sqrt{\Delta\gamma_s d} = \frac{\sqrt{3}}{2}\sqrt{\rho}\sqrt{C_D}\; w \tag{6}$$

$$\frac{\dfrac{U_* d}{\nu}}{\sqrt{\dfrac{T_o}{\Delta\gamma_s d}}} = \frac{\sqrt{3}}{2}\,\frac{wd}{\nu}\sqrt{C_D} \tag{7}$$

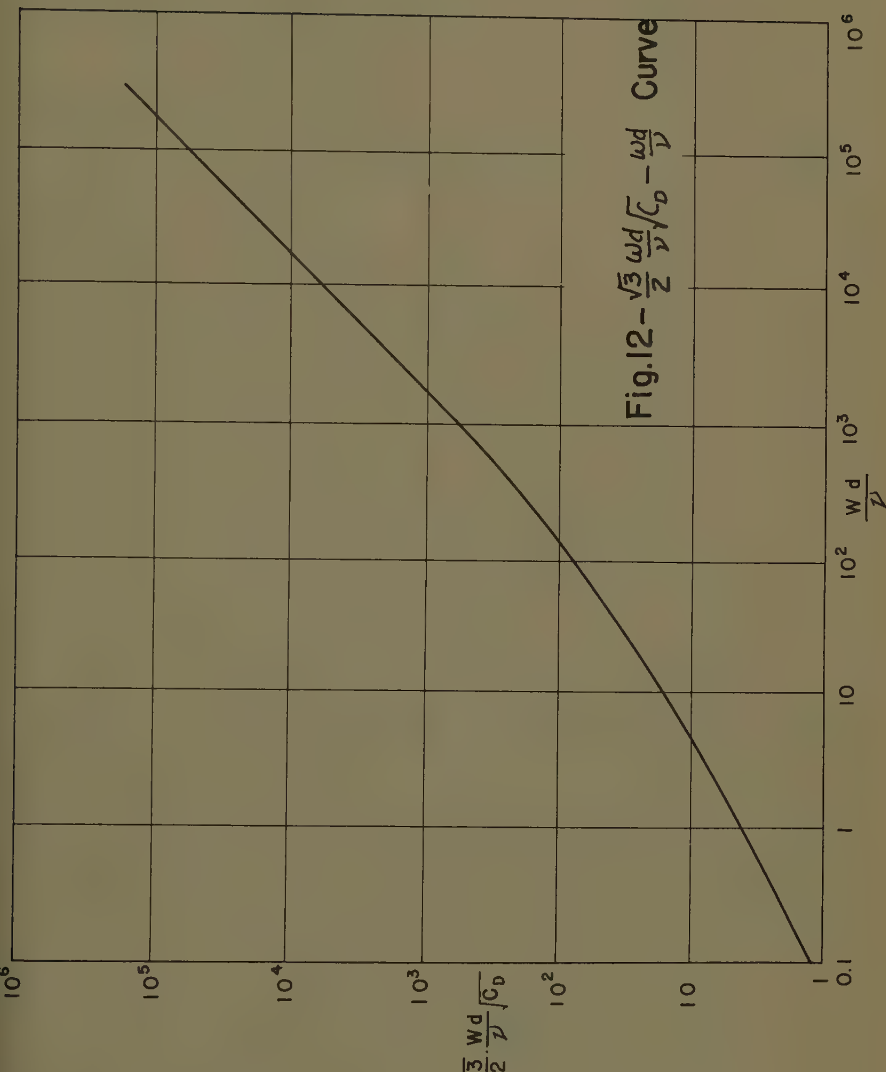


Fig.12 - $\frac{\sqrt{3}}{2}\frac{\omega d}{\nu}\sqrt{C_D} - \frac{\omega d}{\nu}$ Curve

$$\frac{U_*}{w} = \frac{U_* d}{\nu} \Big/ \frac{wd}{\nu} \tag{8}$$

The fall velocity can be expressed as shown in Eq. 5; therefore, Eq. 6 can be obtained accordingly, where C_D is a function of the fall-velocity Reynolds number of the particle, if the shear-velocity Reynolds number is divided by $\sqrt{T_o/\Delta \gamma_s d}$ as shown in Eq. 7, the ratio is a certain function of the fall-velocity Reynolds number (see Fig. 12). Therefore, for a given ratio $U_* d/\nu \Big/ \sqrt{T_o/\Delta \gamma_s d}$, there is a definite value of the fall-velocity Reynolds number. By knowing the fall-velocity Reynolds number and the shear-velocity Reynolds number, U_*/w, the movability number can be obtained by Eq. 8.

Although it has been shown that Eq. 3 and Eq. 4 are interchangeable for spherical particles, it can be demonstrated, however, that Eq. 3 is preferred. If we recall that Shields derived his criterion for the beginning of motion by basing upon the concept of laminar sublayer. According to the concept of laminar sublayer, when the shear-velocity Reynolds number is approximately greater than 70, the laminar sublayer does not have any more effect on the velocity distribution. However referring to Shields' criterion of the beginning of motion, we can find that when the shear-velocity Reynolds number is greater than 70, it still has effect on the coefficient of critical tractive force $\frac{T_c}{\Delta \gamma_s d}$. This fact is a mislead to the concept of boundary layer which was used in the derivation of Shields' criterion. Shields has recognized such a descrepancy, and attributed to the non-quadratic resistance of the grain in the bed. In other words the drag coefficient pertaining to the grain at this stage is still a function of the fall-velocity Reynolds number.

The coefficient of the critical tractive force is changed to movability number U_*/w as shown in Fig. 10. When the shear-velocity Reynolds number is greater than 70, the transposed Shields' curve is independent of $U_* d/\nu$, which is consistant with the theory of boundary layer. This means that when the boundary is fully turbulent (or completely rough), the critical tractive force for the beginning of motion is dependent on the fall velocity of the particle. It implies that the viscosity may still have a slight effect on the determination of the tractive force. For example when the shear-velocity Reynolds number is 70, the corresponding fall-velocity Reynolds number is around 550 of which the drag coefficient is still a function.

To demonstrate further that the coefficient of critical tractive force is not a very suitable parameter for use, we can examine again the Shields' plot, Fig. 13. When the shear-velocity Reynolds number is less than 3, Shields extends his curve to a straight line, where the equation is:

$$\frac{T_c}{\Delta \gamma_s d} = \frac{0.1}{\frac{U_* d}{\nu}} \tag{9}$$

or

$$T_c = \left(\frac{\gamma}{g}\right)^{\frac{1}{3}} \left[0.1\, \nu\, (\Delta \gamma_s)\right]^{2/3} \tag{10}$$

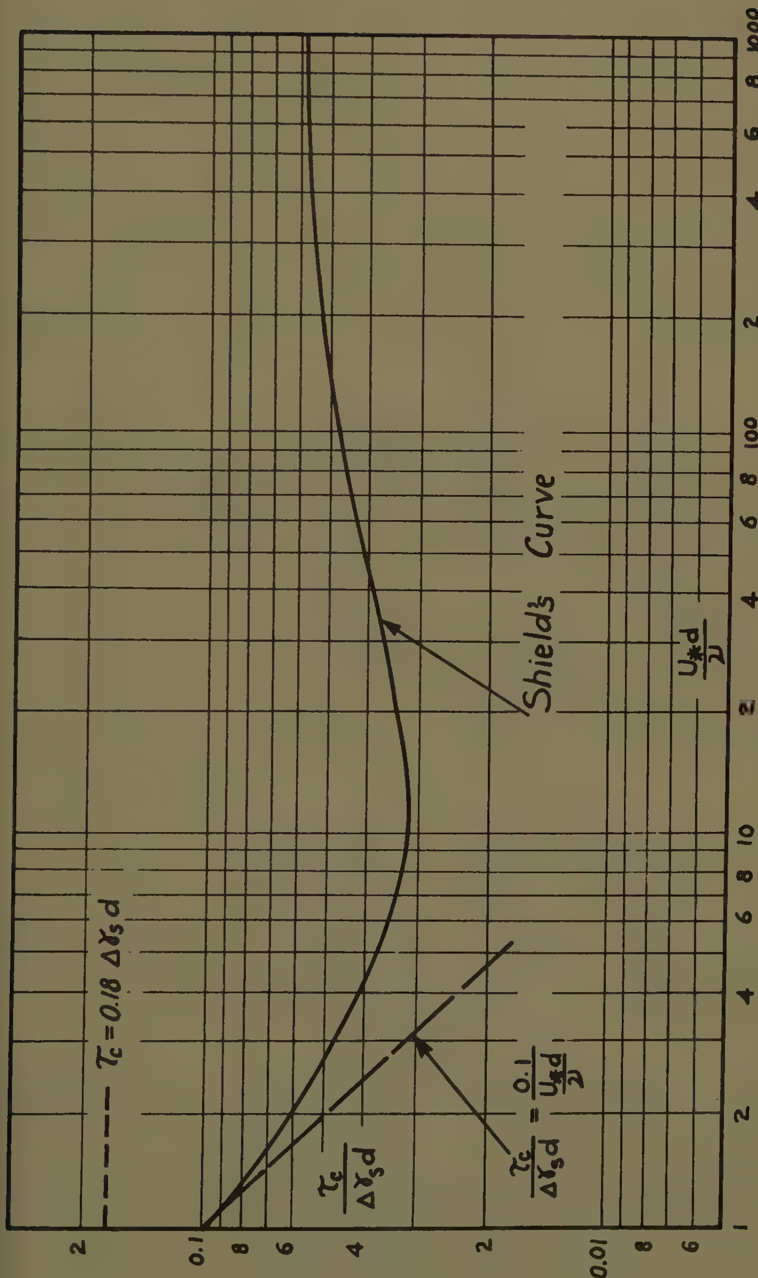


Fig. 13 - $\frac{\tau_c}{\Delta\gamma_s d}$ - $\frac{U_* d}{\nu}$ Curve, Beginning of Motion.

which means when the sediment particle is very small, the shear is independent of the sediment size. However, White[31] and Iwagaki[32] have shown that for small shear-velocity Reynolds number, the critical tractive force is proportional to the sediment size as shown in Eq. 11

$$\tau_c = C \, \Delta\gamma_s \, d \tan\phi \tag{11}$$

where C is a coefficient, ϕ is the friction angle pertaining to the grain. The difference of results between Shields' and White's is very significant, yet cannot be explained according to Fig. 13. If the movability number is used as shown in Fig. 10, the conclusions of White's and Shields' can be obtained separately, depending upon the slope of the extended curve at very low Reynolds number.

If

$$\frac{U_{*c}}{w} = \frac{C_1}{\left(\frac{U_{*c} d}{\nu}\right)^2} \tag{12}$$

Where U_{*c} is the critical shear-velocity
Therefore

$$U_{*c}^3 = C_1 \frac{w \nu^2}{d^2} \tag{13}$$

Within the range of the Stoke's law

$$w = \frac{1}{18} \frac{\Delta\gamma_s d^2}{\mu} \tag{14}$$

Substituting Eq. 14 in Eq. 13 yields

$$U_{*c}^3 = \frac{C_1}{18} \frac{\Delta\gamma_s}{\mu} \nu \tag{15}$$

Therefore

$$\tau_c = \left(\frac{\gamma}{g}\right)^{\frac{1}{3}} \left[\frac{C_1}{18} \Delta\gamma_s \nu\right]^{2/3} \tag{16}$$

the coefficient of o.1 in Eq. 10, corresponds a value of $C_1 = 1.8$.

On the other hand if

$$\frac{U_{*c}}{w} = \frac{C_2}{\frac{U_{*c} d}{\nu}} \tag{17}$$

$$U_{*c}^2 = \frac{\tau_c}{\rho} = C_2 \frac{w\nu}{d} \tag{18}$$

Substituting Eq. 14 into Eq. 18

$$U_{*c}^2 = \frac{\tau_c}{\rho} = \frac{C_2}{18} \frac{\Delta\gamma_s d}{\rho}$$
$$\tau_c = \frac{C_2}{18} \Delta\gamma_s d \tag{19}$$

according to Fig. 10 $C_2 = 1.3$, therefore

$$\tau_c = 0.073 \Delta\gamma_s d \tag{20}$$

The movability number U_*/w is essentially proportional to the square root of the coefficient of tractive force $T_o/\Delta\gamma_s d$; therefore the scatter of data is reduced if U_*/w is used. However as demonstrated above that U_*/w is better than $T_o/\Delta\gamma_s d$ for several reasons. Furthermore, recent evidence by Ippen(30) and Durand(38) also demonstrates that the fall-velocity is a very suitable parameter in the study of bed-load movement. It is not clear however how the fall velocity is a better parameter to be used as far as the mechanics is concerned. Therefore further research on this parameter is urgent.

In order to use Fig. 10 for determining the beginning of motion or the beginning of ripples, it is necessary to use trial and error methods; because the shear velocity is involved in both coordinates. However such a difficulty can be overcome if the shear-velocity Reynolds number is changed as in the following

$$\frac{U_* d}{\nu} = \frac{U_*}{W} \cdot \frac{Wd}{\nu} \tag{21}$$

Therefore Fig. 10 can be changed as shown in Fig. 14 from which the criterion can be obtained very easily if the sediment size, fall-velocity and temperature are known. Similarly Shields' coefficient of critical tractive force is plotted in Fig. 15 according to the corresponding fall-velocity Reynolds number. Comparison of Fig. 15 with Fig. 13 will illustrate why $\frac{T_c}{\Delta\gamma_s d}$ is not a constant when the shear-velocity Reynolds number is greater than 70. At this point the shear-velocity Reynolds number is about 550 which is not the Reynolds number at which the drag coefficient is constant.

Since the change of water temperature is not appreciable under ordinary conditions, the effect of viscosity on the criterion of the beginning of ripples is not very important. For engineering practice the criterion for the beginning of ripples and that for the beginning of motion have been reduced to be a function of the sediment size only as shown in Fig. 16. It is shown that when sediment size is about 2 mm, ripples will not form, although some other type

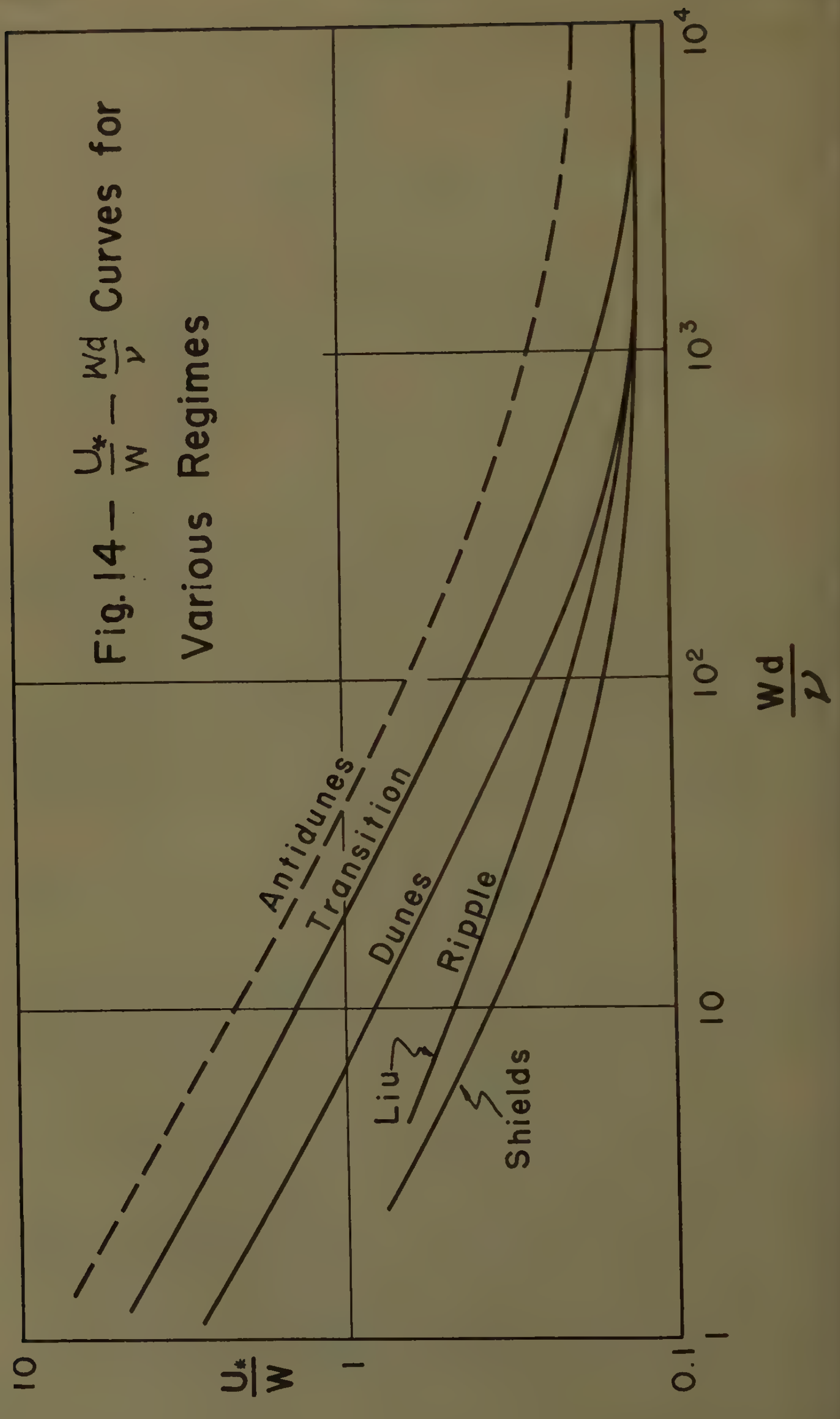


Fig. 14 — $\frac{U_*}{W}$ — $\frac{Wd}{\nu}$ Curves for Various Regimes

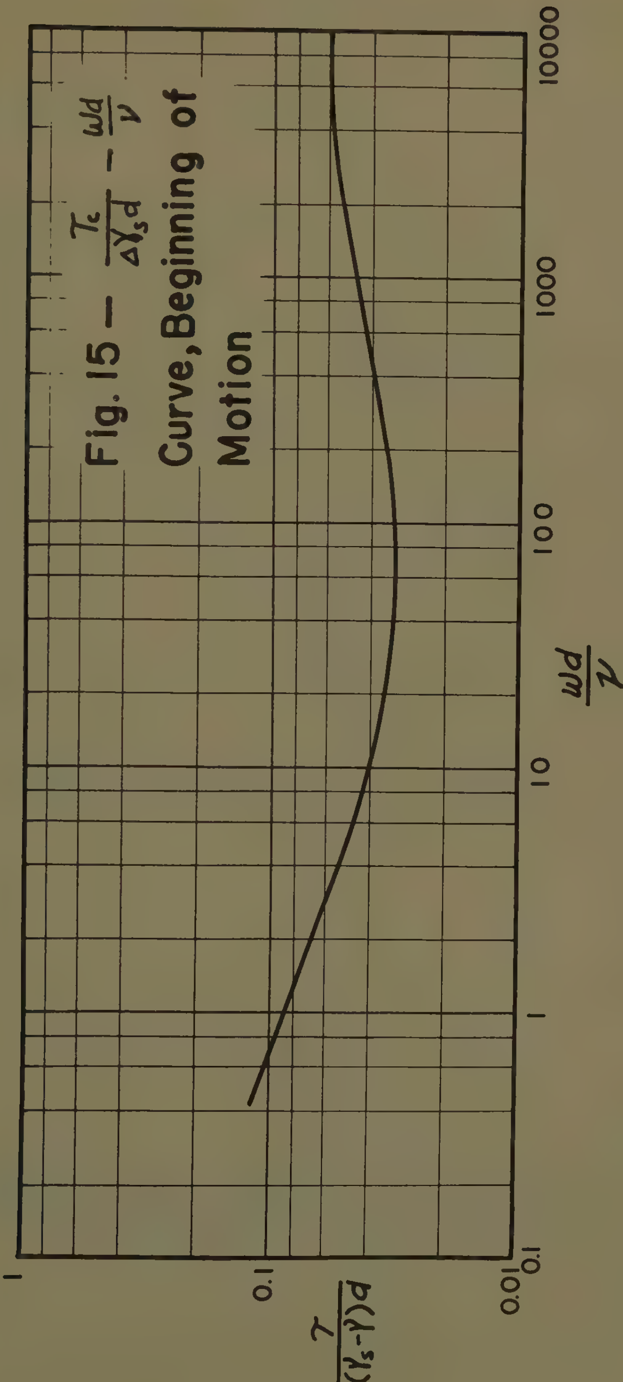
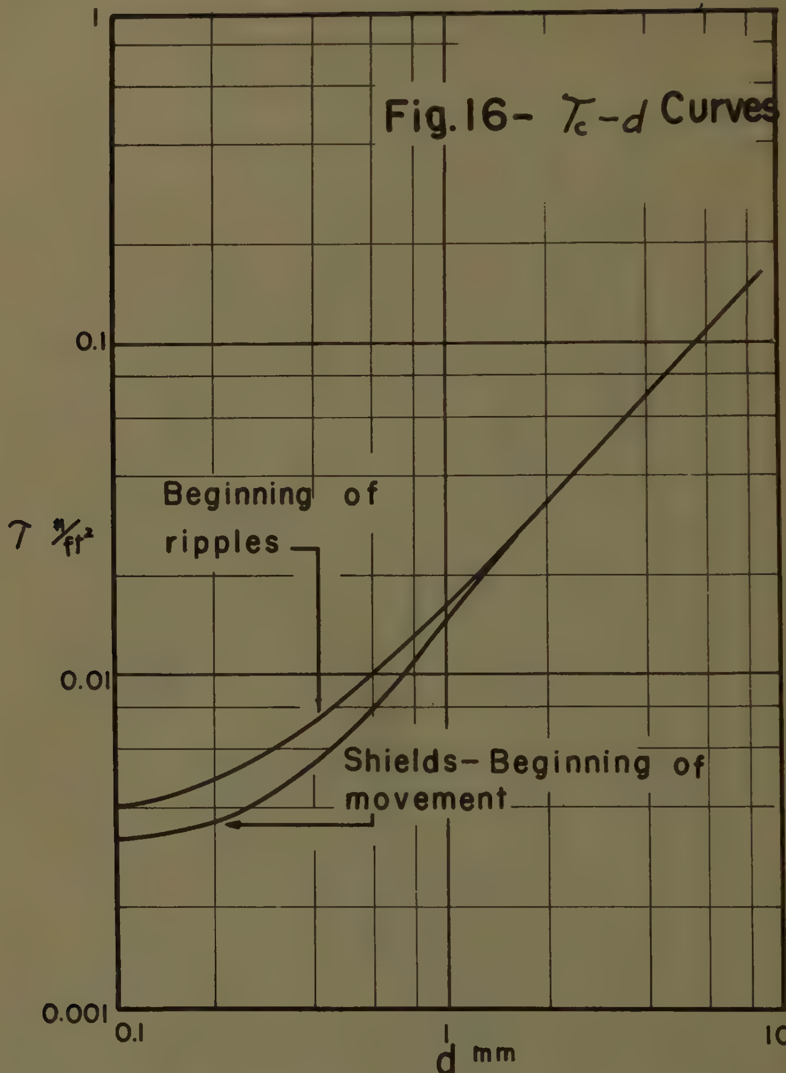


Fig. 15 — $\frac{\tau_c}{\Delta\gamma_s d}$ — $\frac{wd}{\nu}$ Curve, Beginning of Motion



of interface waves may occur when $U_* d/\gamma > 70 \sim 80$ or approximately $d > 2mm$.

Plate,(28) based upon the present knowledge of velocity distribution near the boundary in a turbulent flow and the mechanics of sediment transport, has proposed some emphirical formulas for the beginning of motion and for the beginning of ripples as shown in Fig. 18. The basic procedure of his approach is as follows:

since

$$\frac{U}{U_*} = \frac{1}{K} \ln \frac{y}{y_o} \tag{22}$$

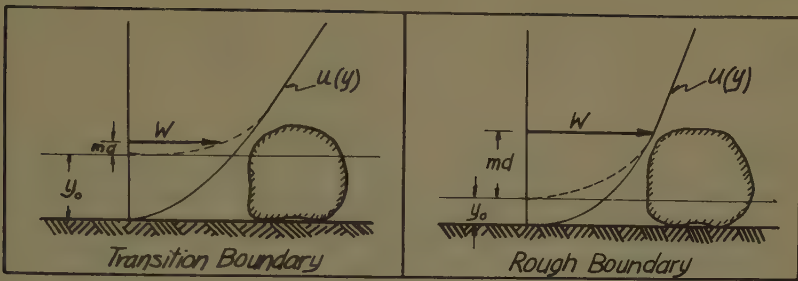


Fig. 17— Definition sketch for Velocity Distribution of a Turbulent Flow Near a Boundary

where at y = y_o, u = 0 (see Fig. 17), and k is the universal constant.

At

$$y = y_o + md, \quad U = W \tag{23}$$

therefore

$$\frac{W}{U_*} = \frac{1}{k} \ln\left(1 + m\frac{d}{y_o}\right) \tag{24}$$

(a) For rough boundary, if the local velocity being equal to w is assumed to be at the grain level, that is

$$md = d - y_o \tag{25}$$

and

$$y_o = \frac{1}{30} d \tag{26}$$

Substituting Eqs. 25 and 26 in Eq. 24 yields

$$\frac{W}{U_*} = 8.5 \tag{27}$$

(b) For transition boundary, m is assumed to be proportional to $U_* y_o / \nu$, i.e.,

$$m = m' \frac{U_* y_o}{\nu} \tag{28}$$

Substituting Eq. 28 into Eq. 24 yields

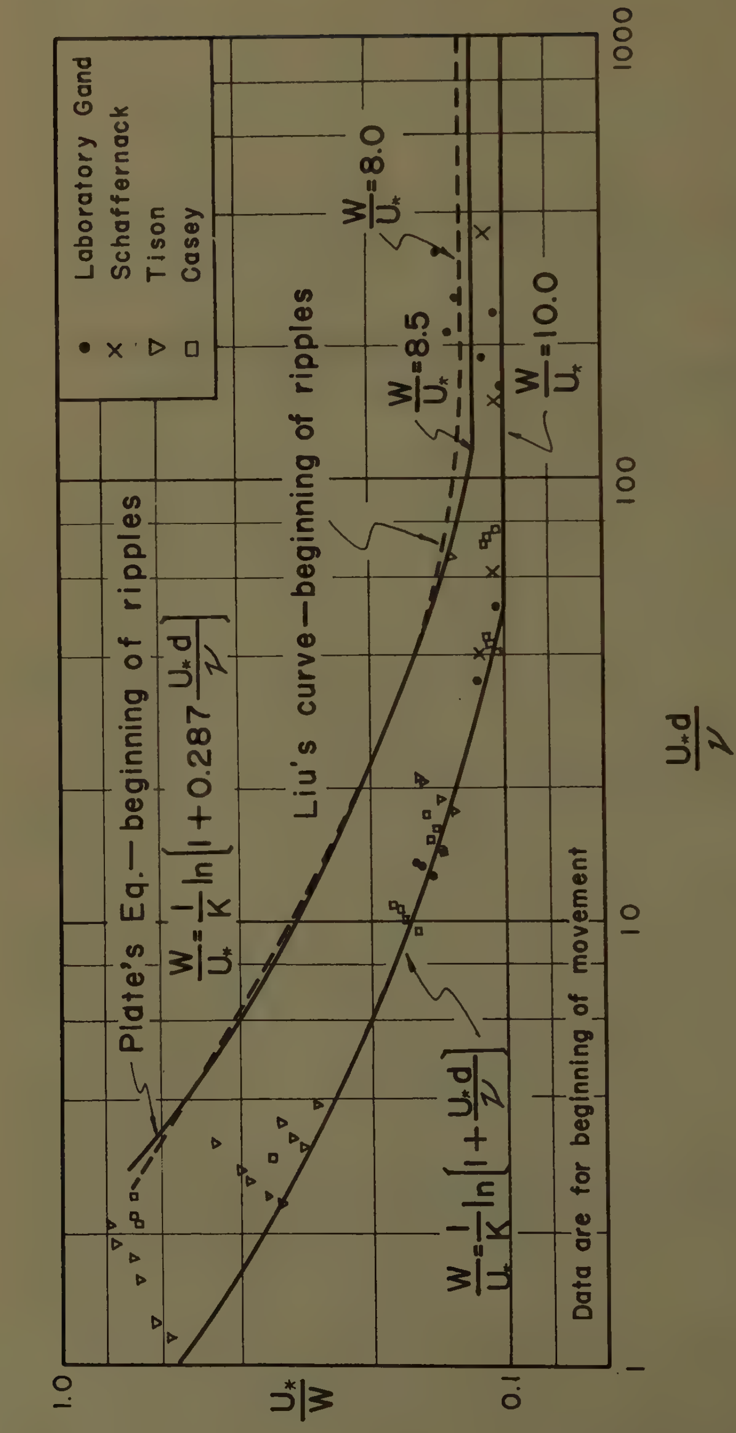


Fig.18 — Plate's Equations for Sediment Motion

$$\frac{w}{U_*} = \frac{1}{\kappa} \ln\left(1 + m' \frac{U_* d}{\nu}\right) \tag{29}$$

Hence based upon the assumption that when the motion of sediment begins, the local velocity at certain level y_0 + md is about equal to the fall-velocity, and using Karman-Prandtl velocity law, Plate found some empirical equations for the beginning of ripples, and the lowest possible case of the beginning of motion as shown in Fig. 18.

The writer agrees with Mr. Maddock that it is important to study the transition of the boundary from hydraulically smooth to hydraulically rough, because it is the prerequisite for the understanding of mechanics of sediment transport, although most of the sediment transport in the laboratory and part of the sediment transport in the natural rivers are connected with this transition region, our present knowledge of fluid mechanics is not sufficient to deal with this phenomenon. Furthermore the writer agrees with Mr. Maddock's statement,

> "The lengthening of the ripple form as the flow changes from laminar to turbulent in terms of the Reynolds number as noted by Shields might be explained in an over-simplified fashion as being due to the changes in the cycle of formation of vortices in the boundary layer."

This simplified fashion well serves the purpose for explaining the mechanics of ripple formation for practical engineers.

Mr. Maddock noticed that in Fig. 10 the ration U_*/w is proportional to $\sqrt{C_D}$; this can be shown as follows

$$\frac{U_{*c}}{w} \propto \frac{1}{\left(\frac{U_{*c} d}{\nu}\right)^N} \tag{30}$$

$$\left(\frac{U_{*c}}{w}\right)^{1+N} \propto \frac{1}{\left(\frac{w d}{\nu}\right)^N}$$

For flow within the Stoke's law

$$C_D \propto \frac{1}{\frac{w d}{\nu}} \tag{31}$$

therefore

$$\left(\frac{U_{*c}}{w}\right)^{1+N} \propto C_D^{\,N} \tag{32}$$

if N = 1,

$$\frac{U_{*c}}{w} \propto C_D^{\frac{1}{2}} \tag{33}$$

This is the case which White stated that the shear is proportional to the size of sediment for the beginning of motion.

On the other hand if N = 2

$$\frac{U_{*c}}{w} \propto C_D^{2/3} \tag{34}$$

which corresponds to the Shields' result.

Mr. Maddock is correct in stating that when there is considerable movement of sediment, the Karman-Prandtl Equation for smooth boundary may not be applicable, because the laminar sublayer might be destroyed due to the sediment movement.

Mr. Tinney questions the meaning when the shear-velocity Reynolds number exceeds 120, the ripples will form as soon as the motion of sediment begins. There seems to be confusion due to the writer's definition of ripples. As shown in Fig. A, dunes can form at $100 < U_*d/\gamma < 400$, at the beginning of motion. Those which form at $U_*d/\gamma > 400$ have not been classified clearly. The reason that interfacial waves will be formed as soon as the motion begins at $U_*d/\gamma > 400$ might be due to other factors such as surface waves which are associated with high values of U_*d/γ. Recently Barton(39) has attempted to classify the bed configuration according to the appearance and geometry of the interfacial waves.

Drs. Vanoni and Brooks have raised some very vital questions pertaining to the writer's paper. These questions are probably most typical in the minds of engineers. Excluding those questions discussed previously, the answers to the remaining questions are as follows.

They question the basic assumptions that (a) a sand bed behaves like a dense fluid, and (b) sand waves and interfacial waves are basically caused by the same fluid forces. The writer recognizes that the reaction of the sand bed is probably not the same as a denser liquid. However, the kind of reaction may be different yet the basic principle, such as Newton's law of motion remains the same. For example, an ideal fluid is non-viscous and a real fluid is viscous, one may state that therefore the study of an ideal fluid flow might be meaningless and should not be linked with a real fluid flow. However, the study of the ideal fluid flow has contributed tremendous progress to those of the real fluid flow. The writer's assumptions in dealing sand waves may be considered over-simplified, nevertheless they may be very useful in the study of sediment motion. The writer emphasizes the points of the similarity; Drs. Vanoni and Brooks emphasize those of dissimilarity.

They question the necessity of using stability theory for deriving the criterion for ripple-formation and propose to use the dimensional analysis. The method of dimensional analysis has been known to hydraulic engineers for some time; however, it is not possible to use the dimensional analysis to get all the answers the engineer wants. In order to use the dimensional analysis, it is necessary to have some knowledge about the physical meaning and the mechanics of the phenomena. Otherwise we will get many unnecessary dimensionless parameters.

They have questioned whether there is an interval between the beginning of motion and that of the beginning of ripple formation. Mathematically it may be difficult to prove that such an interval does exist. It is true that the bed configuration will be very stable or is changing once the ripples are formed. However, the data used by the writer are at the point of beginning of ripples and are not after the ripples have begun. Therefore, the corresponding shear

will not be much more than the incipient shear which is necessary for the incipient motion. Tison(6) has found that sediment may be transported along a plane bed if the flow is laminar. This is another evidence that ripple-formation should not coincide with the beginning of motion. The writer objects to moving the curve for the beginning of ripples down to that of the beginning of movement as suggested by them.

Drs. Vanoni and Brooks do not concur with the writer that dunes will disappear when the flow conditions fall below the curve in Fig. 9. They use the field observation as an evidence, that is, in a natural river if the flow or stage were reduced below the beginning of motion, the dunes would remain unchanged. This situation may happen when the flow is suddenly reduced below the critical stage of sediment movement. However, if the low flow which is capable of transporting sediment is maintained for some long period of time and at the same time all dunes are affected by this low flow, the writer believes that eventually all the dunes and ripples will be smoothed out by the flow. This can be shown in Fig. 9 by the laboratory data of the U. S. Waterway Experimental Station and Kramer's data designated as falling stages. These data are included in Table 3.

They seem to agree that the mechanics of ripple formation can be explained by the theory of Sir Inglis, who favors the theory of turbulence effect. Messrs. Albertson, Simons, and Richardson correctly stated about the effect of turbulence on the formation of ripples,

> "----Such turbulence in turn is directly related to the instability of the laminar sublayer. In other words, turbulence and instability of the laminar sublayer are intimately related and cannot be separated--"

The basic concept of instability also implies that when the eddies caused by the accumulation of sediment are initiated, and when the flow reaches certain instability, such an accumulation of ridges or sand waves will be amplified. On the other hand if the flow does not reach the degree of instability, any such existence of wave-formation will be damped out eventually. Therefore the effect of the eddies behind the sediment grains may act as an instigator, but it is up to the flow near the boundary to amplify or to damp its existence. According to Drs. Vanoni and Brooks, ripples are mainly caused by turbulence; however, they noticed that in the writer's experiments, except Run 1-4, the Reynolds number was rather low. The lack of fully developed turbulence for the ripple formation does not necessarily indicate that the writer's results are opened to question as stated by them, but rather indicates that the theory by Drs. Vanoni and Brooks is questionable.

They think the ripples shown in Fig. 8 should be called antidunes; those of Fig. 6 should be called oblique waves. According to them the reasons that the sand waves of Fig. 8 should be called antidunes are (a) the ripples are round in profile and (b) the corresponding Froude number is 1.3. The writer thinks that the conditions for the formation of antidunes are (a) sand waves moving toward upstream, (b) flow of shallow depth with surface waves (c) high Froude number, (d) fully developed turbulent flow, and (e) constant supply of sediment transported. For the case shown in Fig. 8, there are three of these five conditions missing: (a) ripples did not move upstream, (b) there was no equilibrium supply of sediment and (c) the flow was in the transition region, and was not fully developed. The fact that Fig. 8 shows the round crest of the ripples are not necessarily the conditions of antidunes. These round crested waves can also be caused by undulation of the flow pattern near the bottom.

TABLE IV Data of Exp

Reference	Investigator	Run No.	Sediment Size, mm	Specific Gravity	Temperature °C	Fall Velocity ω cm/sec
34	USWES	1	0.566	2.65	27.0	9.5
34		2	0.541		27.0	8.8
34		3	0.523		16.8	7.6
34		4	0.506		16.0	7.2
34		5	0.483		15.8	6.8
34		6	0.347		20.0	5.0
34		7	0.310		27.0	4.6
34		8	0.205		16.0	2.4
34		9	4.077		19.0	46.0
27		10-5-10*	0.976		15.1	14.5
27		11			14.8	
27		6-41	do	2.65	15.5	14.5
27		42			15.4	
27		7-9	do	2.65	15.9	14.5
27		10			15.9	
27		8-45	do	2.65	17.4	14.5
27		46			17.3	
27		9-9	do	2.65	15.0	14.5
27		10			14.9	
27		10-34	do	2.65	14.5	14.5
27		35			14.6	
27		H_2-20-5	0.908	1.85	25.9	9.6
27		6			26.0	
27		H_3-21-4	1.329	1.74	14.9	12.4
27		5			15.7	
27		22-6	do	1.74	21.0	12.4
27		7			21.0	
27		23-5	do	1.74	22.1	12.4
27		6			22.2	
27		C_1-24-9	3.300	1.35	25.6	16.1
27		10			25.8	
27		C_2-26-4	1.097	1.35	13.5	6.1
27		5			13.5	
27		27-2	do	1.35	10.9	6.1
27		3			11.0	
27		C_3-28-9	3.197	1.32	23.0	15.3
27		10			23.0	
27		C_3-29-6	3.197	1.32	23.0	15.3
27		7			23.0	

*10-5-10 indicates Sand No. 10, Table 5, Run No. 10, etc.

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er ity ec	Av. Sheer Velocity U_* cm/sec	$\frac{\bar{U}_*}{\omega}$	$\frac{d\bar{U}_*}{\nu}$	$\frac{\omega d}{\nu}$	Tractive Force T_b Dynea/cm²	$\frac{T_b}{(\delta s - \delta)d}$	Bed Conditions
	2.23	0.235	15.3	66	4.96	0.0523	
	2.24	0.255	14.2	56	5.01	0.0571	
	2.13	0.281	10.3	38	4.54	0.0535	Average
	2.16	0.300	9.9	34	4.66	0.0569	incipient
	2.01	0.296	8.6	30	4.04	0.0517	ripple
	1.67	0.331	5.8	17.3	2.79	0.0498	formation
	1.70	0.370	6.2	17.3	2.89	0.0573	
	1.50	0.654	2.8	4.5	2.25	0.0660	
	5.99	0.130	250.0	1860	35.9	0.0545	
9	2.44	0.168	20.9	122	5.95	0.0376	Smooth
9							local waves
1	2.54	0.176	21.9	123	6.45	0.0417	local waves
7							Smooth
2	2.61	0.180	22.8	127	6.80	0.0430	Smooth
0							local waves
1	2.58	0.178	23.4	134	6.66	0.0421	local waves
6							Smooth
2	3.01	0.208	25.7	125	9.05	0.0571	Smooth
0							local waves
7	2.65	0.183	22.4	120	7.01	0.0450	local waves
4							Smooth
3	2.25	0.235	23.4	100	5.06	0.067	Smooth
2							waves
5	1.92	0.155	22.7	136	3.69	0.0384	Smooth
9							waves
7	2.25	0.182	30.3	162	5.06	0.0526	Smooth
2							waves
7	2.25	0.182	31.1	166	5.06	0.0526	Smooth
4							waves
2	2.15	0.134	80.5	630	4.62	0.0408	Smooth
3							waves
7	1.54	0.253	14.4	53.5	2.37	0.0632	Smooth
1							waves
7	1.57	0.258	13.6	48.6	2.46	0.0637	Smooth
3							waves
3	2.12	0.136	72.0	504	4.50	0.0448	Smooth
5							waves
5	2.32	0.151	79	503	5.37	0.0535	Smooth
0							waves

TABLE IV Data of Ex

Reference	Investigator	Run No.	Sediment Size, mm	Specific Gravity	Temperature °C	Fall Velocity ω cm/sec
27	USWES	C_4-30-5	1.485	1.32	23.3	8.8
27		6			24.0	
27		31-3	1.485	1.32	23.0	8.8
27		4			23.0	
27		C_6-34-2	1.168	1.31	13.6	6.21
27		3			13.3	
27		35-3	1.168	1.31	14.0	6.21
27		4			11.5	
27		C_8-37-4	1.260	1.26	15.3	6.2
27		5			15.4	
27		R_1-40-5	2.777	1.11	19.7	7.75
27		6			19.8	
27		R_2-41-3	1.317	1.11	15.7	3.76
27		4			15.7	
27		G_1-44-4	3.553	1.07	13.2	5.69
27		5			13.3	
27		G_4-50-1	1.249	1.05	14.0	2.07
27		2			14.0	
27		G_5-51-3	3.112		14.2	3.78
27		4			13.9	
27		52-3	3.112	1.03	14.9	3.78
27		4			15.3	
27	Casey	h-1/200-14	2.455	2.65	20.0	33
27		15				
27		1/400-11	2.455	2.65	20.0	33
27		12				
27		1/800-17	2.455	2.65	20.0	33
27		18				
27	Jorissen, A.L.	I-I-2	0.720	2.67	22.2	11.5
27		3			22.2	
27		I-I-4	0.720	2.67	22.8	11.4
27		5			20.5	
27		II-II-14	0.930	2.67	22.8	15.0
27		15			23.3	
27		II-II-17	0.930	2.67	22.8	15.0
27		18			20.0	
27		II-II-18	0.930	2.67	20.0	15
27		19			21.1	
27		II-II-23	0.930	2.67	20.5	15
27		24			20.5	

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eer ocity U_* /sec	Av. Sheer Velocity U_* cm/sec	$\frac{\overline{U}_*}{\omega}$	$\frac{d\overline{U}_*}{\nu}$	$\frac{\omega d}{\nu}$	Tractive Force T_b Dynes/cm^2	$\frac{T_b}{(\delta s - \delta)d}$	Bed Conditions
.40	1.46	0.166	23.7	120	2.13	0.0457	Smooth
.52							waves
.54	1.62	0.184	25.6	125	2.62	0.0562	Smooth
.70							waves
.36	1.45	0.234	14.3	53	2.10	0.0593	Smooth
.53							waves
.51	1.60	0.258	16.0	56.1	2.56	0.0723	Smooth
.69							waves
.52	1.58	0.255	17.6	57	2.50	0.0856	Smooth
.65							waves
.10	1.16	0.150	30.7	170	1.35	0.0450	Smooth
.23							waves
.94	1.01	0.269	11.9	37.8	1.02	0.0719	Smooth
.08							waves
.03	1.05	0.185	31.1	200	1.10	0.0403	Smooth
	1.07						waves
.76	0.85	0.411	6.2	10.1	0.72	0.118	Smooth
.93							waves
.61							Smooth
.69							waves
.90	0.80	0.212	21.9	85	0.64	0.0726	Smooth
.00							waves
.60	4.70	0.142	96	665	22.1	0.054	Smooth
.79							lt ripples
.70	3.74	0.113	765	7300	14.0	0.034	Smooth
.78							lg waves
.94	4.01	0.122	82	665	16.1	0.039	Smooth
.09							lt waves
.19	3.23	0.281	24.4	87.5	10.4	0.087	Smooth
.27							ripples
.40	3.16	0.277	23.4	85.5	9.97	0.083	ripples
.93							Smooth
.94	2.99	0.200	29.7	148	8.93	0.058	Smooth
.03							mod ripples
.30	3.00	0.200	28.8	142	9.00	0.058	Smooth
.71							ripples
.71	2.94	0.196	27.6	140	8.64	0.0556	Smooth
.17							sml ripples
.94	2.97	0.198	27.8	138	8.81	0.0567	Smooth
.00							sml ripples

TABLE IV Data of Experi

Reference	Investigator	Run No.	Sediment Size, mm	Specific Gravity	Temperature °C	Fall Velocity ω cm/sec
27	Chyn, S.D.	3-I-IV-1	0.847	2.65	22.5	13.6
27		2			21.2	
27		10	0.847	2.65	22.2	13.6
27		11			21.8	
29	Kramer	I	0.530	2.70	12	7.4
29		II	0.510	2.70	12	7.1
29		III	0.550	2.70	12	7.7
29			0.550	2.70	12	7.7
29			0.550	2.70	12	7.7
27	Boragardi, Yen	I-6-5	10.56	2.63	11.0	95.2
27		7			9.5	95.2
27		6-21	10.56	2.63	15.0	95.2
27		22			15.0	
27		II-8-57	7.12	2.64	19.0	64
27		59			19.0	
27		8-61	7.12	2.64	19.0	64
27		-63			19.2	
27		8-91	7.12	2.64	19.8	64
27		92			19.6	
27	Liu, T.Y.	I-17f-26	4.4	2.66	26.5	49
27		-27			26.5	
27		17e-6	4.4	2.66	30.0	49
27		7			29.0	
27		II-17i-8	3.4	2.66	28.5	42
27		9			28.5	
27		17j-5	3.4	2.66	27.5	42
27		6			27.0	
27		III-17l-13	2.3	2.66	30.5	33
27		14-14			30.5	
27		17m-6	2.3	2.66	29.5	33
27		7			29.5	
27		IX-17p-8	1.4	2.66	25.5	22
27		9			25.5	
27		V-17v-9	3.6	2.66	24.5	44
27		10			24.5	
27		VI-17w-12	1.8	2.66	26.5	27.5
27		13			26.5	
27		17x-11	1.8	2.66	26.5	27.5
27		12			26.5	
27		17y-7	1.8	2.66	24.0	27.5
27		8			24.0	

on Ripple-Formation -- Con't.

eer city $_*$ sec	Av. Sheer Velocity U_* cm/sec	$\frac{U_*}{\omega}$	$\frac{d\bar{U}_*}{\nu}$	$\frac{\omega d}{\nu}$	Tractive Force T_b Dynea/cm^2	$\frac{T_b}{(\delta s - \delta)d}$	Bed Conditions
83	2.89	0.212	25.5	119	8.35	0.0595	Smooth
95							dune 0.5cm
91	2.82	0.207	25.0	120	7.95	0.0566·	Dune 0.5cm
73							Smooth
	2.13	0.288	9.01	52.5	4.54	0.0534	
	2.22	0.313	9.00	50	4.93	0.0405	
	2.31	0.300	10.05	61	5.34	0.0472	
	2.27	0.295	10.1	35.5	5.15	0.0563	
	2.46	0.320	10.8	34.7	6.05	0.0661	
63	11.14	0.117	905	6000	124	0.0725	Frly Smooth
65							scour
73	12.02	0.127	1.110	6700	145	0.0857	Smooth
30							Scoured
40	8.51	0.134	590	4150	72.4	0.0643	Smooth
62							Scoured
72	9.07	0.142	647	4250	82.1	0.0730	Frly Smooth
43							Scoured
92	9.23	0.144	650	4210	85.1	0.0756	Smooth
55							rough
70	5.77	0.118	294	2400	33.3	0.0465	Smooth
84							gen ripples
87	4.70	0.096	255	2550	22.1	0.0309	Smooth
52							local ripples
61	4.67	0.111	192	1670	21.8	0.0395	Smooth
72							lt local rips.
83	4.79	0.114	192	1630	22.9	0.0415	Smooth
75							local ripples
45	4.40	0.133	128	950	19.4	0.0516	Smooth
34							lt local rips
64	3.77	0.114	107	920	14.2	0.0379	Smooth
90		0.150					local ripples
25	3.31		52.5	347	11.0	0.0482	Smooth
37		0.124					light dunes
33	5.47		218	1670	29.9	0.0511	Smooth
60		0.151					local ripples
05	4.01		83.5	540	16.1	0.0548	Smooth
98		0.135					local ripples
67	3.72		77.5	560	13.9	0.0471	Smooth
78		0.128					dunes
48	3.52		69.3	520	12.4	0.0422	Smooth
56							local ripples

According to the writer, ripples can be formed by various kinds of disturbances. It is only when the flow reaches the stage of instability that the ripples can be maintained. Fig. 6 probably shows that ripples were caused by the side wall at the entrance. Therefore it does not contradict the writer's explanation of ripple formation.

It is interesting to notice that earlier Dr. Brooks(40) stated that the shear force or the shear velocity cannot be used in the study of sediment transport, because of the change of bed configuration. In his discussion of Fig. 9, he and Dr. Vanoni propose to use shear velocity in the dimensional analysis of ripple formation. In correspondence to the writer, Dr. Brooks stated that the shear velocity is a suitable parameter for studying the low rate of sediment transport up to dune regime, but it is not suitable for flow having flat bed (a transition from dune to antidune) or antidunes.

The discussion by Messrs. Albertson, Simons and Richardson can be considered as an excellent continuation of the writer's paper on the mechanics of ripple-formation. By the use of Fig. A, most of the types of bed-configuration can be estimated. Therefore, the classification of flow in alluvial channel has been clarified considerably. The space is not available here to emphasize all the important contributions. A complete understanding of Fig. A is recommended for those who are interested in the hydraulics of alluvial flow. The writer only brings up the points which need to be discussed excluding those already mentioned earlier.

In Fig. A, there are three kinds of abscissa used. Their relationship can be stated as follows:

assume
$$\delta' = 11.6 \frac{\nu}{U_*} \qquad (11)$$

$$\frac{d}{\delta'} = \frac{1}{11.6} \frac{U_* d}{\nu}$$

since $R_e = \frac{VD}{\nu}, \quad r_o = D/2, \quad \sqrt{f} = \frac{U_*}{V}\sqrt{8} \quad \text{and} \quad k = d$

therefore
$$\frac{R_e\sqrt{f}}{r_o/k} = 4\sqrt{2}\,\frac{U_* d}{\nu}$$

Considering that there is considerable information presented on Fig. A already, for the sake of clarity we need not use different kinds of abscissa. Furthermore, the definition of the thickness of the laminar sublayer δ' is rather arbitrary, for example, Rouse has reported $\delta' = 11.6\ \nu/U_*$, Rotta(40) defines $\delta' = 6.83\ \nu/U_*$, and according to Einstein and Li(42) that the laminar sublayer cannot be well defined. Furthermore, the ratio of sediment to the thickness of the laminar sublayer does not have physical meaning in case of dune bed, therefore the parameter U_*d/ν seems to be preferable.

In order to apply Fig. A for estimating the required shear for certain kinds of bed configuration, a trial and error method has to be employed. This step can be avoided if Fig. A is replotted accordingly U_*/w against wd/ν as shown in Fig. 14. The parameter wd/ν is also known to be the parameter pertaining to the straight line shown as for constant sediment size, if the fluid viscosity and shape factor of the sediment remain constant.

The criterion for the formation of antidunes is doubtful, because the Froude number is very important for antidune flow. Therefore, more laboratory data on the formation of antidunes are needed.

Through private correspondence with the writer, Dr. Bogardi made some very interesting comments on the discussion presented by Messrs. Albertson, Simmons, and Richardson. He thinks that the discussion completes the writer's paper very well. With regard to Fig. A, he stated:

> "For constant temperature and constant d, the nature of regime is defined by the values of U_*.
>
> If U_* is assumed as the factor which quantitatively defines the configuration of the bed, the missing third parameter may be expected to depend uniquely upon the water depth D, the slope, as well as on the particle size d, or upon the combination of the above three quantities. A logic combination of these three quantities is the channel stability factor d/DS introduced in Hungary as early as 1942 to sediment investigation. This factor is proportional to gd/U_*^2, the inverse of the shear-velocity Froude number of the particle U_*^2/gd.

Dr. Bogardi indicated that along the straight line of d = constant in Fig. A, the bed configuration changes as the channel stability factor changes.

From Fig. A, and by assuming a temperature of 20° C, Dr. Bogardi has reduced the criterion of various bed configuration into a series of empirical equations in the form

$$U_* = \alpha \, d^{\beta} \tag{35}$$

as shown in Fig. 19, where α and β are determined empirically.

The suggestions for future research made by Drs. Albertson, and Simons, and Mr. Richardson are mainly for studying the variation of bed-configuration. The writer is particularly interested in seeing item (5) being studied in the near future. Namely, the effect of the drag coefficient of the particle on the mechanics of sediment movement. It has been shown in the previous discussion that the correlation of data by introducing the fall-velocity becomes less scatter and more logic, and from dimensional analysis, the effect of particle shape can be incorporated in the analysis if both the fall-velocity W and the intersity of the submerged weight of the particle $\Delta\gamma_s d$ are used.

Dr. Bogardi pointed out that the parameters used by the writer are similar to those proposed by Lane and Kalinski. By reviewing the works of Lane and Kalinski,(42) the writer could not find the similarity between the factor P and the shear-velocity Reynolds number U_*d/ν. The factor P has been defined by Lane and Kalinski as the ratio of the average sediment-concentration in the vertical to the concentration at the zero-level, and is a function of U_*d/ν and n/d 1/6, where n is the Manning's roughness factor.

His suggestion to discuss various parameters related to the sediment transport is very inspiring. Those parameters related to the mechanics of ripple-formation have been discussed earlier in connection with dimensional analysis. A general discussion of various parameters is intended by the writer in connection with other phase of sediment transport at a later occasion.

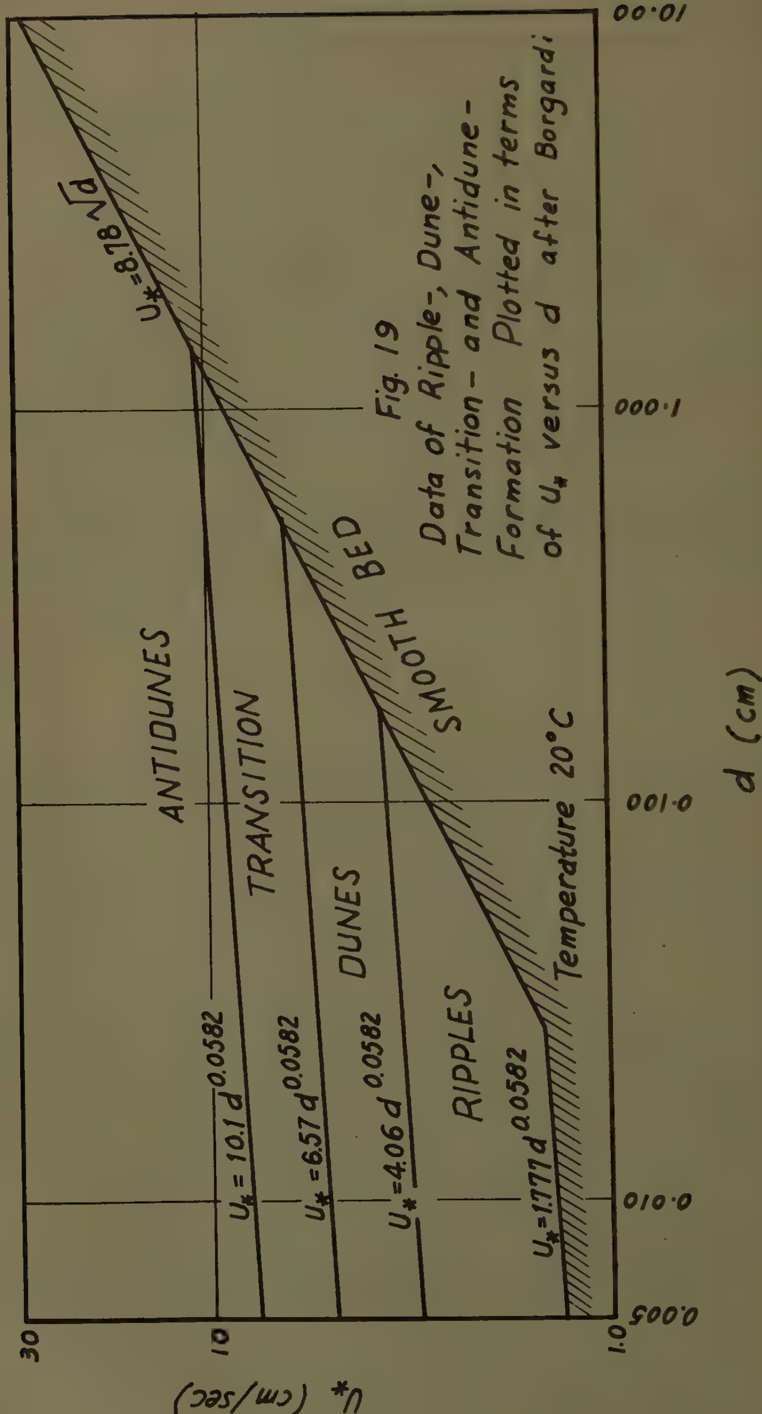

Fig. 19
Data of Ripple-, Dune-, Transition- and Antidune- Formation Plotted in terms of U_* versus d after Borgardi

REFERENCES

27. Johnson, J. W. "Laboratory Investigations on Bed-Load Transportation and Bed Roughness," SCS, TP 50, March 1943.

28. Plate, E. J. "Studies of Sediment Ripple Formation," M.S. Thesis, Colorado State University, Fort Collins, Colorado, 1957.

29. Kramar, H. "Sand Mixtures and Sand Movement in the Fluid Models," Trans. ASCE, Vol. 100, 1935.

30. Ippen, A. T. and Verma, B. P. "The Motion of Discrete Particles Along the Bed of a Turbulent Stream," Internal Assoc. of Hyd. Res. Minneapolis, Minn., 1953.

31. White, C. M. "The Equilibrium of Grains on the Bed of an Alluvial Channel," Royal Soc., London, Proc. Sec. A, 174:322-338, 1940.

32. Iwagaki, Y. "Hydrodynamical Study on Critical Tractive Force," Trans. JSCE No. 41, Tokyo, Japan, 1956. (in Japanese)

33. Barton, J. "A Study of Roughness and Sediment Transport in Alluvial Channels," Ph. D. Dissertation, Colorado State University, Fort Collins, Colorado, 1958.

34. U. S. Waterway Experiment Stations. "Studies of River Bed-Materials and their Movement," paper 17, January 1935.

35. Report No. 4. "Methods of Analyzing Sediment Samples," A Study of Methods used in Measurement and Analysis of Sediment Loads in Streams. U. S. Engineers, St. Paul, November 1941.

36. Wuest, W. "Beitrag zur Entstehung von Wasserwellen durch' Wind," Zeit fün Ang. Math. unt Mech . . 29 (7/8) 239-52, 1949.

37. Smoltczyk, H. U. "Beitrag zur Ermittlung der Feingeschiebe-Mengenganglinie." Berlin, Technische Universitat. Iakultat fur Bauingenieur Wesen. Institut für Wasserbau. Mitteilungen, no. 43:1-62, January 1955.

38. Durand, R. "Basic Relationships of the Transportation of Solids in Pipes - Experimental Research," Proc. Minn. Inter. Hyd. Conv. 1953.

39. Brooks, N. H. "Mechanics of Streams with Movable Beds of Fine Sands," Proc. 668, ASCE, April, 1955.

40. J. Rotta. Das in Wandnahe gultige Geschwindigkeit-s gesetz turbulenter Stromungen, Ingenieur Archiv, 18 Band, 1950. S277-280.

41. Lane, E. W., and Kalinski, A. A. "Engineering Calculation of Suspended Sediment" Trans. AGU, 1941, P. 606.

42. Einstein, H. A., and Li, H. The Viscous Sub-layer along a Smooth Boundary. Am. Soc. of Civil Engineers, Proc. 82: No. 945, E. M. 2:1-27, April, 1956.

43. Bogardi, J. L. "Enquires into Channel Stability and Sediment Movement." Research Institute of Water Economy, Budapest, 1957.

A HIGH HEAD CAVITATION TEST STAND FOR HYDRAULIC TURBINES[a]

Closure by W. G. Whippen and G. D. Johnson

W. G. WHIPPEN[1] and G. D. JOHNSON,[2] A.M. ASCE.—The writers appreciate the high quality of the several discussions, especially the significant differences in viewpoint that were raised.

The repeatable accuracy of $\pm$ 1/4% parallels the writers' previous experience with a well-designed pump-turbine test stand in that it also applies to a unit that has been completely dismantled and reinstalled at a later date . . . . . even with a completely new runner casting of identical design and finish. Although this cavitation stand does not lend itself to tests with controlled air content such as that at the California Institute of Technology, it is possible to determine whether or not the extremes of high or low air content have any measurable effect upon performance.

Professor Hooper should note that the model of Figure 5 shows even higher efficiency than that of Figure 4, which had been increased some 25% in capacity without increasing overall dimensions. Since all instruments have been calibrated accurately in place or under installation conditions, it is felt that these results are accurate to within 1% or less. The difficulties in obtaining accurate field test results have been pointed out and are freely admitted. The establishment of a reliable efficiency step-up formula will require equally accurate field tests, or the rational analysis suggested by the late Dr. Knapp.

Mr. Vander Boegh will understand that, as a result of the commercial requirements for our testing facilities, it has not been convenient to test a completely homologous (or identical) model in both the high and the low head cavitation stands. However, almost identical Kaplan units indicate an efficiency about 2% higher in the new stand. This is attributed to a poor venturi location and difficult alignment conditions in the older stand.

It was stated in the paper that the primary function of the new stand is the testing of homologous models of prototype installations under prototype conditions. The high test heads and resulting high speeds require a special shaft that does not lend itself to the oil head and other equipment required for the determination of Kaplan blade operating forces. This type of testing, as well as visual observation, is carried out with sufficient accuracy in the older low head stand.

Results to date indicate that water temperature does not affect cavitation test results as long as the corresponding vapor pressure is used in the

a. Proc. Paper 1201, April, 1957, by W. G. Whippen and G. D. Johnson.
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formula for sigma, but we must admit that commercial testing requirements have precluded any conclusive basic research in this as well as other areas of interest. The 100-ton cooling system permits the testing of models at the water temperatures expected to prevail at the prototype installations.

Although it was not emphasized in the paper, the head tank was sized to permit installation of a complete model inlet structure for units in semi-spiral case with short intakes; a modeled length of three or four diameters with a bell-mouth intake is installed upstream to reproduce the inflow conditions to a full spiral case.

As mentioned in the paper, a separate pump-turbine test stand has been in operation for more than ten years. The construction of a new high-head pump-turbine stand has been authorized recently; upon its completion it will be possible to carry out test programs in both stands concurrently, if desired.

Naturally, we were pleased by the late Dr. Knapp's agreement that accurate laboratory tests on completely homologous models under prototype conditions could be substituted for less accurate, expensive and time-consuming field tests. We agree with him that the model and the prototype drawings should be identical . . . . . except for size . . . . . and that both the model and the prototype should be manufactured in accordance with the drawings . . . . . within reasonable tolerances, as is our usual practice. In view of the fact that a sincere attempt has been made to duplicate prototype conditions, the use of field testing techniques seems quite appropriate to us. As pointed out in the paper, we are interested primarily in accuracy and simplicity; more elaborate instrumentation will be developed in due course.

It does not seem unusual to us, in spite of the number of people who suspect otherwise, that we have found no significant changes in the calibrations of venturi meters over a period of years . . . . . provided that they are in first-class condition when calibrated and are maintained properly.

The writers would like to mention that cavitation has two important manifestations: (1) rough operation accompanied by reduced efficiency and output and (2) "pitting" or removal of metal from critical areas of the runner. Operation at critical sigma does not reduce efficiency and/or output significantly, but it could produce noticeable cavitation pitting damage over a period of time, unless the critical areas have been protected by stainless steel or some other resistant material.

The writers hope that this paper, together with the points raised by the discussers, constitutes a useful addition to the technology on hydraulic laboratory practice. In closing, we would like to point out that the increasing use of laboratory tests as contract acceptance tests indicates the desirability of a universally accepted test code for models.

THE EFFICACY OF FLOOR SILLS UNDER DROWNED HYDRAULIC JUMPS[a]

Closure by Ahmed Shukry

AHMED SHUKRY,[1] M. ASCE.—The writer wishes to thank all discussers of the paper for their contributary comments or adverse criticism. In fact, it was conceived that the paper would receive due discussion because the conclusions drawn are opposite to some of the common-practice procedures for the following points:

a- Laboratory scour tests may not give definite results when the jump formed below a river barrage is deeply submerged.
b. The best location of an anti-scour sill is not at the floor edge, but rather upstream of that edge.
c- The hydraulic advantages of dentated sills over solid ones, having the same peripheral dimensions are slight.

Long before the present tests were carried out, points b and c were raised by the writer himself in a discussion to the results obtained by the Hydraulic Research Laboratory of the Delta Barrages, Egypt. This was in 1945 when the old Esna Barrage was under re-construction according to the final scheme shown in Fig. (I-f). Sand-scour tests on a model of this barrage showed that when the floor sill was placed at the middle section of the solid floor, the best results were obtained. This conclusion was surprising to engineers working on the design of regulators and barrages in Egypt. It had been the general practice to locate anti-scour sills at the floor edges, or even far downstream from those edges for more safety, as shown in Figs. (I-c) and (I-d).

In 1947, the design features of the new Edfina Barrage were under investigation. Scour tests on a model of this barrage were also carried out in the Delta Barrages Laboratory. The results of these tests were similar to those of Esna Barrage regarding the better functioning of mid-floor sills. Guided by the fresh results of Esna and Edfina models, the sills of some of the minor regulators in Egypt were constructed at their mid-floor sections. This new practice which was starting to replace the old common one of using edge sills raised considerable discussion in 1948. The tests of Esna and Edfina barrage models were condemned although the two years of actual experience with the reconstructed Esna Barrage showed that with a midfloor sill, the downstream movable bed was practically scourless. Following the same testing procedure of sand-scour tests, which is explained in the paper, new tests were made on the same model of Edfina Barrage and in the same laboratory. The results of

a. Proc. Paper 1260, June, 1957, by Ahmed Shukry.
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these latter tests, which are presented in Fig. (2) in the paper, were more surprising due to their complete lack of agreement with the first ones. This contradiction of the test results added more complexity to the problem, and the location of the sill in the prototype, which was under construction, was postponed until other tests, with other experimental technique, would be carried out in another laboratory.

The first thing the writer did was to review the results of most of the laboratory scour tests which had been performed on the Nile Barrages and major Egyptian regulators since 1928. It may be of interest to give a general description of the main features of this type of structures! Nile Barrages are open dams with vents varying between 5.0 and 8.0 m in width (16.40 and 26.24 feet) and piers varying between 2.0 and 2.5 m in thickness (6.56 and 8.20 feet). The working head on the barrage is generally about 4.5 m (14.76 feet). The river bed material being a mixture of silt and sand, extending to considerable depths, a certain length of solid apron is always required downstream of the piers to provide safety against percolation. The tailwater depth is much bigger than the conjugate depth corresponding to the flow issuing from the vents according to equation (1), and a deeply drowned jump is always formed on the downstream solid floor. The hydraulic conditions of these structures are, therefore, different from those of most of the American spillway dams, where perfect jumps are usually formed on their downstream aprons, and for which the Froude number is the main influencing parameter of the flow. The adhering property of the drowned jump is that it does not dissipate as much energy as that of a perfect jump. The more the jump is drowned, the more will be the part of the excess energy which is to be dissipated by viscous action downstream of the jump. Accordingly, the Reynolds parameter may become as much important as that of Froude. This explains the inconsistency of some of the previous scour results as pointed out in the paper.

From the discussions by Mr. Peterka and Dr. Leliavsky, it seems that there is a general misconception of the conclusion given in the paper regarding the validity of sand scour tests. It is only when the jump is deeply drowned that the results of these tests may become uncertain. The writer wishes to write the suggestion of Dr. Leliavsky as follows: For important structures operated under deeply drowned jumps, velocity tests should be used to check any uncertainty in the results of scour tests. In fact, this procedure was followed in the design of Edfina Barrage. However, since the object of the velocity tests is to predict the erosive tendency of water, the tests should be made on models with fixed beds. Velocity patterns recorded in a movable-bed model may not indicate the actual erosive forces in water because part of the excess energy may have been dissipated in forming the scour pool.

When the jump is perfect or slightly submerged, there will be only one effective parameter. In this case qualitative similarity of scour tests is feasible. Professor Vito A. Vanoni and the writer have recently used scour tests to design the optimum proportions of some of the bed stabilizers required for some of the flood control channels in Southern California. The report of these tests is not yet published.

Mr. Thomas raised a question about the difference in action of an end sill when the jump is drowned and when the jump is clear, provided that in both cases measures are taken to normalize the flow upstream of the floor end. The writer believes that if such normalization has taken place by the provision

of sills, baffle piers or blocks, there will be no need for the addition of an edge sill in both cases of jumps. Such an addition will not secure more safety to the floor, but on the contrary, an edge sill may cause re-disturbance of the normalized velocity distribution and may give rise to bed shearing forces due to the momentum changes at the floor edge. This is clear in Figs. (IIa) and (IIb) of the paper.

The statement given by the writer that "The efficiency of any sill against bed erosion is indicated by the rate of adjustment of the flow to the normal distribution in the downstream channel," is not an assumption as commented by Mr. Peterka but it is rather a definition of the efficiency of a floor sill. This sill is intended to deflect the high-velocity currents upwards in order to establish normal distribution of velocities within the shortest length from the pier edges.

The scour hole is mathematically defined in equation (2) of the paper just to illustrate the various parameters involved in the problem of drowned jumps, under which conditions qualitative similarity between models and prototypes may not be feasible. The writer agrees with Mr. Peterka that identical erosion patterns may be obtained for the prototype, if models of the same structures are constructed with different scales, and if the same bed material is used in all models. Such testing procedure may be, however, very long and expensive.

The results of the present tests show that the improvements gained by the flow when using toothed sills instead of solid ones of the same peripheral dimensions, are relatively small compared with the expenditure of maintaining toothed sills against the more severe cavitation actions.

The writer agrees with Professor Ingersoll that due to the part of the excess energy which is dissipated by viscous action downstream of the model, differences between the model and the prototype values of $\beta 2$ may be expected. Accordingly, the values of τo, as computed by equation (6), may be less for the model than for the prototype. However, due to the relatively short length of the solid apron over which the normal velocity distribution is established, the expected differences in the values of τo may be small.

The application of the energy equation between Stations I and VII for the conditions presented in Figs. (6a), (7a) and (7b), as given by Professor Ingersoll, is an illuminating example which illustrates numerically the wake effects of the piers. Due to this wake, the energy losses or the shear stresses at the bed cannot be numerically determined between successive sections, unless the distribution of velocities in the lateral direction has been also recorded. In the present work, qualitative analyses of various designs was only required. This was achieved by velocity measurements along the flume centerline.

From his experience in West Pakistan on works similar to Egyptian barrages and regulators, Mr. Mushtaq Ahmad advocates that velocity distribution and comparative studies of scour should be taken as criteria for the proper design of scour prevention devices. Mr. Ahmad states the general conclusions of his very interesting experimental study on inclined weirs of the "Glacis" type. Fig. (I-c) in Mr. Ahmad discussion shows that for a deeply drowned jump, the depth of the scour pool is relatively larger than what the velocity distribution diagrams might indicate. This may explain the writer's point of view that velocity distribution studies should be carried out on models with fixed beds. Velocity distribution, recorded after the excess energy in water has been expired in scour, gives no indication of what might take place in nature.

The discussion by Mr. Ahmad under the heading "Mechanics of Energy Dissipation in the Stilling Basin and Function of Scour Control Devices" is most interesting. It throws much light on the hydraulic jump as an energy converter.

Questions were raised by Mr. Thomas and Mr. Peterka about the data to which Figs. (8) to (II) relate. In all tests, the discharge was passed below the lower gate. The super-critical depth of the flow between the piers was 1.20 m (3.94 ft.), while the tailwater depth was 4.50 m (14.76 ft.). The barrage was assumed to work under a hypothetical head of 4.00 m (13.12 ft.). The maximum rate of discharge through the three barrage vents was fixed from the flood conditions and was taken as 302 cubic m per sec. (10670 cubic ft. per sec.). The width of one vent is 8.00 m (26.24 ft.) and the thickness of one pier is 2.50 m (8.20 ft.). The length of the downstream solid floor, excluding the block extension, is 20.00 m (65.62 ft.). All these dimensions relate to the prototype.

In closing, the writer wishes to mention that although research on erosion problems have started earlier than most of the works in other problems of hydraulic engineering, there are still many aspects in the scour phenomenon requiring further analyses. Rationalization of the old empirical rules in the light of modern fluid mechanics seems to be still lacking.

CORRECTION.—Page 6 in the paper, conversion of 302 cubic ms. per sec. yields 10670 cubic ft. per sec., rather than 106700 cubic ft./sec.

The fifth reason presented has to do with the better dissipating action of a slightly drowned jump. While this is true, the point is somewhat academic when applied to a large flood control or multi-purpose reservoir project with basin designed for a discharge three times or so larger than any previously recorded flood.

With regard to abrasion damage caused by bed material or other debris circulating on the apron, the writer agrees with Mr. Peterka that "dead" areas should be eliminated and that ground rollers can sometimes be too strong.

Mr. Peterka's theory concerning the cause of damage at Lucky Peak Dam is an interesting one. Whether the damage was caused by impinging jets or by vortices (the model did not indicate either), the writer agrees that further investigation is desirable.

In conclusion, it was hoped that this paper would contribute to the profession a little more knowledge regarding the difficulties and shortcomings of actual prototype hydraulic structures and to some extent the possible limitations of hydraulic models. It was further hoped that the paper would serve to open discussion of experiences of others in this field.

STILLING BASIN EXPERIENCES OF THE CORPS OF ENGINEERS[a]

Closure by R. H. Berryhill

R. H. BERRYHILL,[1] A.M. ASCE.—Since this paper was presented, a rare pportunity to observe stilling basins in action presented itself in the form of ie Southwest Flood of 1957. This flood ranks as one of the six most destruc-ve floods in America since 1900. At least 30 dams of the Corps of Engi-eers were affected to varying degrees in Texas, Oklahoma, Arkansas, ansas, and Louisiana. Many of the spillways in this area were operated for ie first time during this flood. The experiences encountered here and in everal other parts of the country recently should probably be told in a sub-equent paper.

With regard to the discussion by Mr. Peterka, which incidentally was very uch appreciated by the author, comments in the following paragraphs are ffered.

The first reason given by Mr. Peterka for believing that a basin floor iould be set at full conjugate depth or more is that the safety of the entire ructure may hinge on whether the baffle piers remain fully effective after prasion or erosion. It should be noted that the model tests conducted by the orps for hydraulic-pump-type basins invariably include tests with the baffle iers removed to insure that spray action does not occur even with the ex-remely rare flow termed the spillway design discharge. An additional safety ctor lies in the fact that even on those projects where some rounding and prading of the baffle piers has occurred over the years, the resulting stilling fect of the baffle piers certainly could not be termed "ineffective," although is granted that some reduction in their efficiency had taken place. Neither the two examples given by Mr. Peterka were the result of reliance on baf-e piers.

The second and third reasons given by Mr. Peterka concerns the future egradation of the river channel and lowering of the tailwater as well as the ossible incorrectness of the tailwater curve. It should be noted that for ajor projects, considerable investigation precedes the determination of the ilwater curves, both present and ultimate degraded. A safety factor is in-uded in the determination of the latter, and the stilling basin design is made lequate for both tailwater conditions.

The fourth reason presented has to do with the lag of actual tailwater epth during periods of increasing discharge. With the usual large flood-ontrol project, this feature is usually of little consequence whereas on a in-of-river project it is sometimes of major importance. Discretion in this atter during the planning stage of such projects is absolutely necessary.

Proc. Paper 1264, June, 1957, by R. H. Berryhill.

Chief, Design Branch, U. S. Army Engineer District, Albuquerque, N. Mex.

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With regard to abrasion damage caused by bed material or other debris circulating on the apron, the writer agrees with Mr. Peterka that "dead" areas should be eliminated and that ground rollers can sometimes be too strong.

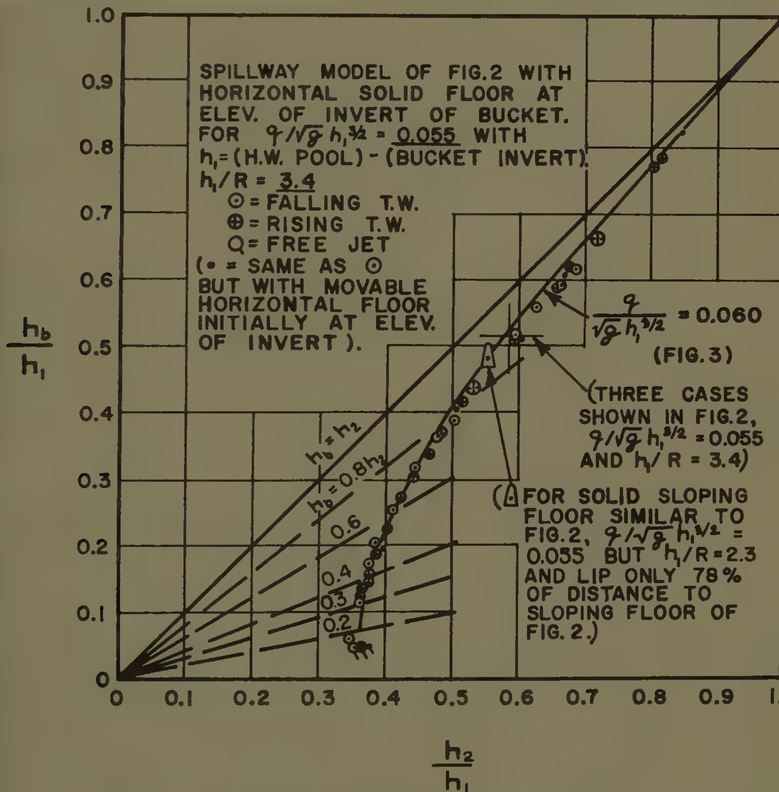
Mr. Peterka's theory concerning the cause of damage at Lucky Peak Dam is an interesting one. Whether the damage was caused by impinging jets or by vortices (the model did not indicate either), the writer agrees that further investigation is desirable.

In conclusion, it was hoped that this paper would contribute to the profession a little more knowledge regarding the difficulties and shortcomings of actual prototype hydraulic structures and to some extent the possible limitations of hydraulic models. It was further hoped that the paper would serve to open discussion of experiences of others in this field.

A STUDY OF BUCKET-TYPE ENERGY DISSIPATOR CHARACTERISTICS[a]

Closure by M. B. McPherson and M. H. Karr

M. B. McPHERSON,[1] A.M. ASCE and M. H. KARR,[2] J.M. ASCE.—After the paper had been completed it was realized too late for rectification, that all



EFFECT OF RISING AND FALLING TAILWATER

FIGURE C-1

a. Proc. Paper 1266, June, 1957, by M. B. McPherson and M. H. Karr.
1. Research Engr., Philadelphia Water Department, Philadelphia, Penna.
2. Investigations Engr., State Water Resources Bd., Salem, Oregon.

test runs had been made with a falling tailwater. Thereupon a special series of tests was conducted using the Greensboro model shown in Figure 2, with a horizontal solid floor at the elevation of the bucket invert. The results of these tests, for a q-parameter of 0.055 and an h_1/R of 3.4, are plotted in Fig. C-1. The free-jet definitely occurred at an h_b less than 0.2 h_2 with a falling tailwater, but with a rising tailwater roller action was not unequivocably established until h_b was about 0.3 h_2.

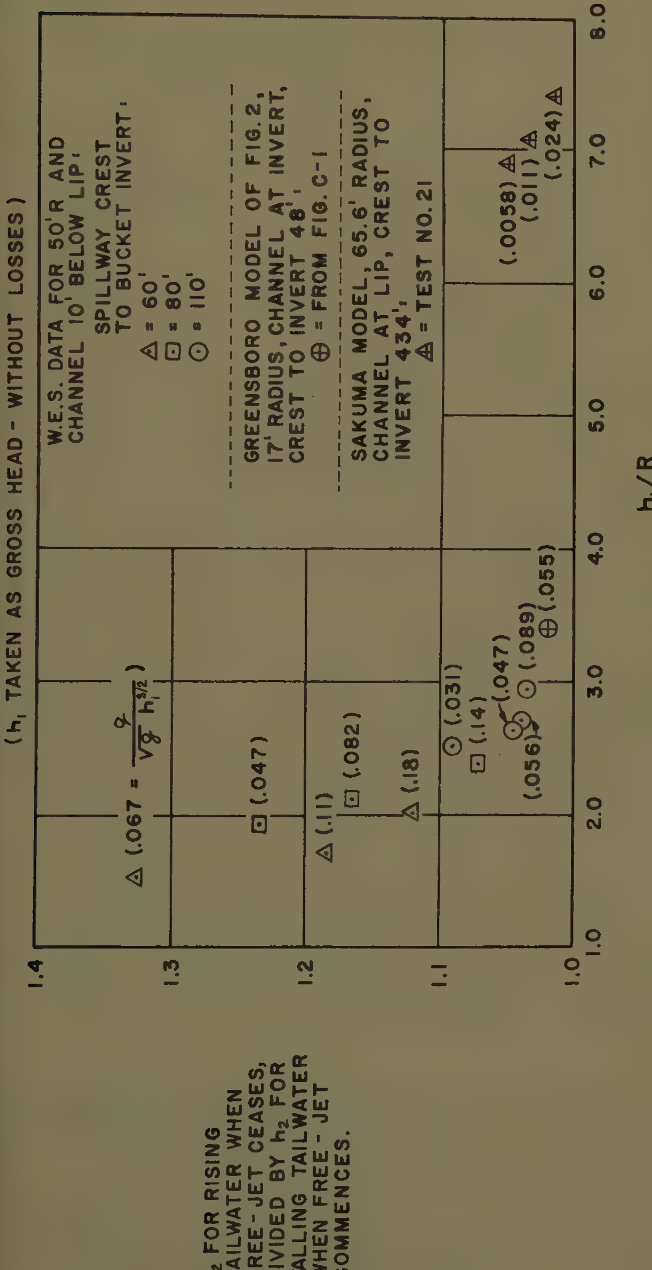
Plotted in Figure C-2 is the ratio of the h_2's for the free-jet limit with a rising and falling tailwater versus h_1/R for the tests of Fig. C-1, comparable tests on the Sakuma Dam model[1] and for unpublished design data from the Army Engineer's Waterways Experiment Station. The data points are indorsed with the corresponding q-parameter values. The comparative tailwater requirements appear to be a function only of h_1/R, but this limited evidence is hardly conclusive.

The statement given under "Recommendations for design": - - "Free-jet performance can be expected whenever h_b is less than 0.2 h_2, in general," is therefore applicable to a falling tailwater only. Due to the small changes in h_2/h_1 at an h_b less than about 0.3 h_2, and considering the limited evidence presented in Figure C-2, a limit of h_b equals 0.3 or slightly higher is indicated for h_1/R greater than about 3 to marginally avoid a free-jet with a rising tailwater. This tentative conclusion diminishes somewhat the competitive status claimed for the solid bucket over a hydraulic jump in a horizontal channel (See Figure 5).

The data presented by Mr. Dougherty from the Weiss Dam model studies do not and should not agree with the graph of Fig. 3 for several reasons. The head above the crest is as much as 41.5% of h_1. Mr. Dougherty recognizes the probable effect on the data by the resultant roller distortion in stating that "This is attributed to the effect of the flow over the crest." Because of the spillway crest piers the flow width at the crest is only 83.5% of the bucket width. Only the exit angle of 45^o should be compared with Fig. 3. (The data for the modified 45^o exit is not identified; no correlation for a non-radial exit should be expected). The q-parameter curves for 0.12 and 0.15 are Mr. Dougherty's extrapolations. Whether or not they are representative is difficult to say. The three curves from Fig. 3 in his graph have been accurately reproduced. Since the h_1/R ratios for the 45^o exit represented a good range (1.9, 3.1 and 5.1), it is unfortunate that the Weiss model characteristics were so dissimilar as to preclude a proper comparison with the data by the authors.

It is not clear why Mr. Dougherty is hesitant to accept locating the downstream bed at the elevation of the bucket invert. The only reason the sloping floor tests represented in Fig. 2 were given was to show that despite sloping the bed beyond a point where the surge roller action is dissipated, the relation of h_2 in terms of h_b is unaffected even with moderate modifications of the bed adjacent to the bucket. In Fig. C-1 is shown a plotted point for an h_1/R of 2.3 (25'R rather than 17'R of all other points) for which the 41' horizontal portion of the solid floor of Fig. 2 was reduced to 32' through replacement with the larger bucket in the original model tests. The slight displacement of this data point could be attributed to a small degree of crowding of the expanding surge by the nearer sloping bed. The data for the three cases detailed in

1. "Hydraulic Model Tests on Flood Spillway of Sakuma Dam," by Atsuya Okada and Takeshi Ishibashi, Central Research Institute, Electric Power Industry, Japan, June 1, 1956.



FREE - JET RISING AND FALLING TAILWATER CHARACTERISTICS
SOLID BUCKET - SIMPLE RADIAL LIP - 45° EXIT.

FIGURE C - 2

Fig. 2 are in good agreement with the solid and movable bed horizontal floor data of Fig. C-1.

In the Sakuma Dam model tests[1] it was necessary to provide a substantial secondary dam downstream from the bucket to insure adequate tailwater depths. When this dam was placed too close to the bucket, the surge expansion and the general action was disrupted considerably. For a certain critical position, and beyond, the action above and at the bed was unaffected and unchanged.

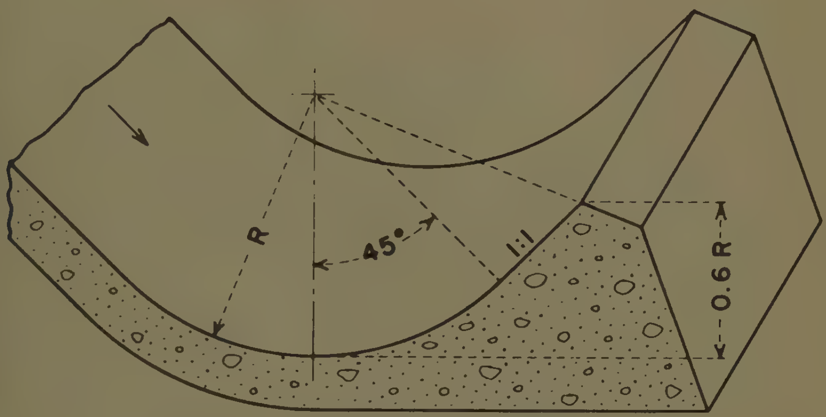
Mr. Dougherty's statements concerning erosion are entirely true. However, in the low to medium head flood control and water supply spillways for which the study data is particularly suited, scouring below the main structure can often be tolerated. The worst scour would occur at the design head, which is of course rarely attained. To answer the question on bed material size, the 1:40 scale Greensboro model bed material was "No. 1-B aggregate, light on No. 8 with an analysis as follows: 100% passing a one-half inch sieve, 75-100% passing a 3/8", 10-30% passing a No. 4 sieve and less than 1% passing a No. 8 sieve." It is quite likely that prototype erosion would be more extensive with a continuous horizontal bed than with a downstream sloping bed similar to the one shown in Fig. 2.

In partial answer to Mr. Dougherty's question concerning setting wall heights lower than h_s, the training walls of both the Greensboro and Penn Forest Dams were set below h_s with a rip-rapped berm immediately above the walls in the splash area. How much the walls can be lowered is related to the position of the end of the wall relative to the surge location, and to the bed and foundation conditions beyond a longitudinal wall in the case of a short training wall. A training wall must be at least high enough to contain h_b. It appears doubtful if training walls could be lowered much more than 2/3 to 4/5 of h_s without causing transverse surge action with a consequent disruption of the bucket roller.

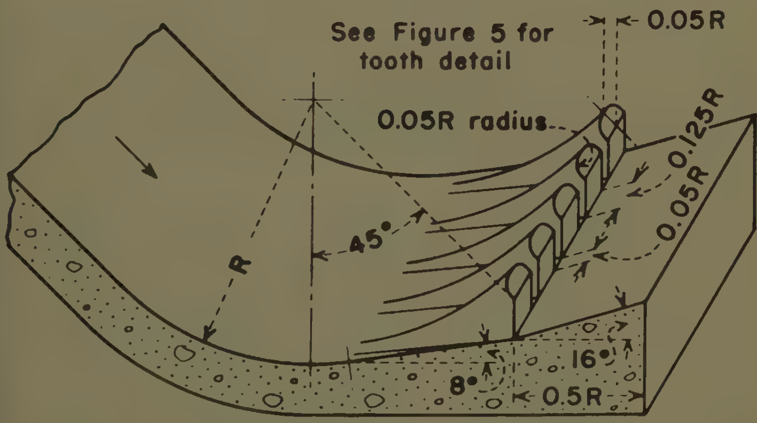
The Sakuma Dam 1:90 scale model tests[1] were performed with a maximum q-parameter of around 0.03 and an h_1/R of between about 6.0 to 6.7 (using the net h_1). Only one series of runs were made with an R of 17 1/2" but 46 series were performed with an R of 8 3/4". Unfortunately, because of a failure to detail the many tests, it is not possible to correlate test results with test conditions except in a few isolated instances. The authors, in consideration of the above and after careful review, have failed to find any sound basis for the interpretation of Mr. Elevatorski that "Investigations by Okada and Ishibashi substantiate the functional relationship given by (Mr. Elevatorski's) equation (3)---." The authors could not locate the quotation alluded to in the remainder of the sentence. The authors have failed repeatedly in attempts to identify those overall test results which were for a 45° radial exit without extensions or modifications of the lip. The approach slope was constant at about (10 on 8).

The U. S. B. R. has made tests on a slotted type of bucket[1] as shown in Fig. C-3, a reproduction of a page from their report. The report acknowledges fully the difference between the solid type and slotted type bucket: "The hydraulic action and the resulting performance of the two buckets are

1. Bureau of Reclamation, Progress Report III, Research Study on Stilling Basins, Energy Dissipators, and Associated Appurtenances, Section 7, Slotted and Solid Buckets for High, Medium and Low Dam Spillways, Hydraulic Laboratory Report No. Hyd.- 415, Denver, July 1, 1956.



A. GRAND COULEE TYPE SOLID BUCKET



B. ANGOSTURA TYPE SLOTTED BUCKET

FIG. C-3: REPRODUCTION OF FIG. I, PAGE 3, FROM U.S.B.R. HYDRAULIC LABORATORY REPORT NO. HYD.-415, JULY I, 1956.

quite different." All the data presented appears to be for a falling tailwater according to the manner in which the results were tabulated. Test results reported include only "seeepout" conditions (same as free-jet), and "diving flow" conditions, with no data for any conditions in-between. In the slotted bucket tests, at a higher tailwater, a point is reached where the jet leaving the bucket is depressed downwards with consequent severe scouring action; this phenomenon is probably impossible of occurrence with a circular solid bucket. The values of h_b are not given for the tests. In order to compare performance, values of h_2/h_1 and the q-parameter were computed from their tabulations of test results and plotted in Fig. C-4, using the headwater pool waterlevel to bucket invert as h_1 (gross head). Also shown are lines of h_b in terms of h_2 taken from Fig. 3, for comparison. Apparently the slotted bucket requires a smaller h_2 to form a free-fet with a falling tailwater. The "diving flow" regime appears to be a serious restriction. The ranges of variables tested with the slotted bucket appear in Table C-1.

In the U.S.B.R. report are mentioned (no test data) tests with three solid buckets with a conventional radial 45^o exit. The h_1/R tested was only about 1.5, 2.4 and 3.6 at a maximum q-parameter of around 0.10 for which the head on the crest was about 27% of h_1.

Mr. Elevatorski's equations (3) and (6) are modifications of the relations set forth on pp. 39-45 in the U.S.B.R. report. The report relations are interpretations based solely upon the somewhat limited tests with a slotted type bucket, which appear to be inconclusive in many respects.

It is difficult to see why Mr. Elevatorski would prefer to interpret the required minimum tailwater and minimum bucket radius for a solid bucket on the strength of results from radically different slotted bucket tests. Moreover, the ranges of variables represented in Table 1 and Table 2 are much more comprehensive than those in Table C-1. The authors feel that the minimum bucket radius requirements for a solid bucket have been fairly well defined through their tests. The solid bucket tests mentioned in the U.S.B.R. report were obviously too fragmentary to permit any generalized comparison with those for a slotted bucket.

Mr. Elevatorski's interpretation of the U.S.B.R. report in stating that the slotted bucket approach slope does not, "within ordinary limits, affect the bucket size" is unfounded since only one approach slope was used in the tests.

Table 1 provided by Mr. Elevatorski is a welcome contribution, but is not by any means a complete inventory. Two of the Greensboro Dam figures given in his table are incorrect: the maximum discharge per foot of bucket width is 137 cfs/ft. and the maximum gross head is 58 feet, rather than 540 and 72.

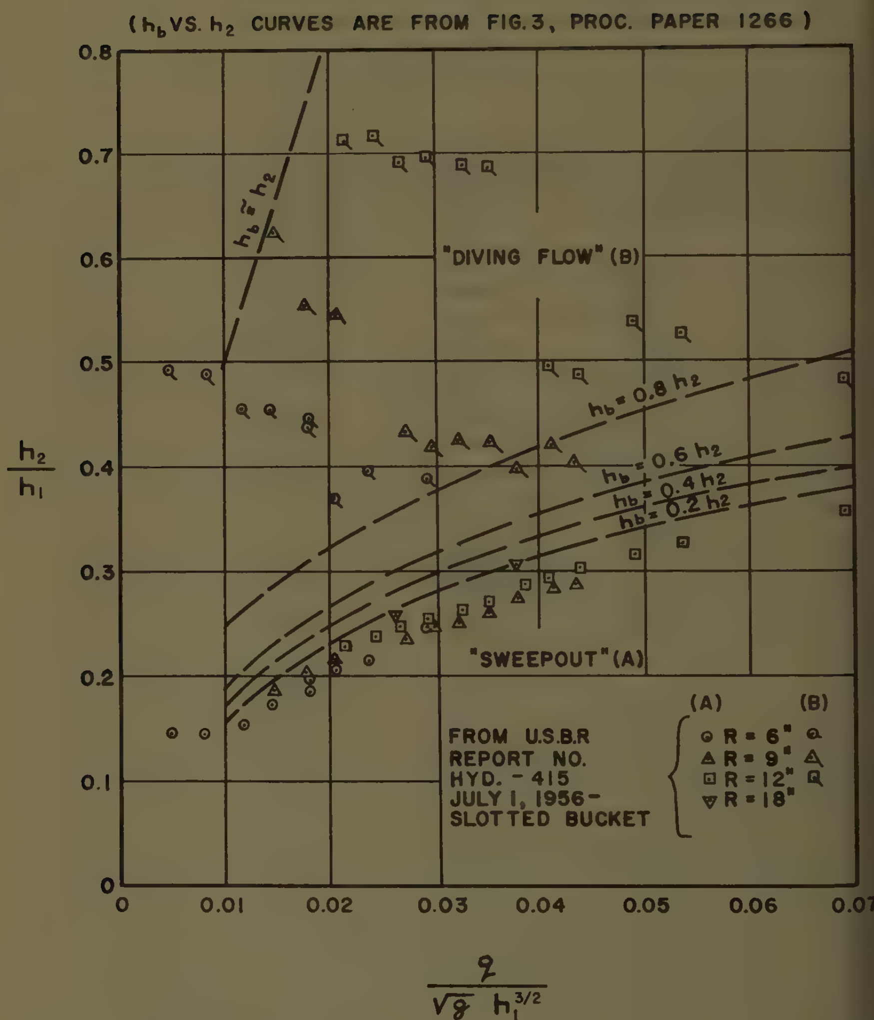
The discussions have contributed one very important point: Comparison with the authors' results cannot be made unless the comparative tests are for a solid bucket with a radial exit (without exit modifications), flow uniformly distributed over the channel width and with the height from the spillway crest to the bucket invert being more than about 75-80% of h_1. Modifications of the exit may have profound effects upon performance.

Acknowledgement is made to Mr. Fred R. Brown for providing the authors with the W. E. S. data, to Mr. Elevatorski for a copy of reference d and to Mr. S. H. Poe of the U.S.B.R. for a copy of reference e. Also, Mr. Elevatorski discovered a serious omission in an explanation given in the original draft, in the form in which it was presented in the ASCE Pittsburgh Convention on October 18, 1956.

TABLE C-1

NGES OF VARIABLES INVESTIGATED WITH AN ANGOSTURA TYPE SLOTTED
CKET WITH (10 ON 7) ENTRANCE AND 45° MODIFIED EXIT.
sing gross head for h_1). U.S.B.R. REPORT[e]. (See Figure C-4).

Radius R, Feet	Range of h_1, Feet	Range of q, cfs/ft.	Range of $\frac{q}{\sqrt{g}\, h_1^{3/2}}$	Range of h_1/R
0.50	5.20 to 5.68	0.31 to 2.25	0.0046 to 0.029	10.4 to 11.4
0.75	5.42 to 5.88	1.05 to 3.52	0.015 to 0.043	7.2 to 7.8
1.00	5.54 to 6.22	1.58 to 6.08	0.022 to 0.069	5.5 to 6.2
1.50	5.63 to 5.80+	2.00 to 2.99+	0.026 to 0.038+	3.8 to 3.9+



SLOTTED BUCKET VS. SOLID BUCKET CHARACTERISTICS

FIGURE C-4.

SYSTEMATIC CHANGES IN THE BEDS OF ALLUVIAL RIVERS[a]

Closure by Walter C. Carey and M. Dean Keller

WALTER C. CAREY,[1] M. ASCE and M. DEAN KELLER,[2] J.M. ASCE.—The writers are gratified by the interest aroused by their paper. They are also pleased with the nature of the comments by Messrs. Blench, Vanoni, and Liu and the evidence that they have reported phenomena in a field relatively unexplored and one that may have an important bearing on river hydraulics. The writers hope that it will be possible to explore this field further, in a more systematic way. The remarks below deal with each of the discussions in turn:

T. Blench.—The observed rate of travel of bed waves of 250 feet per day is highly significant as a rough indication of the possible quantity of material that may be moved in this way. It may be possible to initiate, on the lower Mississippi, a set of sand wave observations conformable with the requirements set forth by Mr. Blench.

Vito A. Vanoni.—As to the increase in size of dunes with increase in stage, the writers were vaguely aware that "stage" was more important than "velocity." Although unable to explain why, they think it reasonable, and by the same token, they think it reasonable that the "increased velocity" in Mr. Vanoni's flume, without any increase in stage, resulted in decreased size of dunes. The writers agree on the desirability of similar sand wave observations at flood stage; in fact, "all around the calendar" and through all variations in stage. Such observations might also throw light on what causes the rating curve at a given station to be a "loop." The writers think it possible that the sediment actually forming the waves is not measureable by any existing procedure. All other sediment, they suspect, may not be too important in governing the hydraulic properties of a stream. They would not be surprised to find that "waves" at different stages may be constituted of somewhat different materials (as to size).

Mr. Hsin-Kuan Liu.—Concerning the transport of bed load around a bend, the writers are prepared to find (when the matter is studied) that bends have higher transporting power than crossings; in fact, that may be why there are "crossings." As to the remark ". . . bed configuration is not an independent variable. . . . ," the writers suspect it is controlled by the river itself for reasons that may be understood some day as a result of further study. This concept includes all systematic variations in the river bed with stage, not only as to systems of sand waves, and heights of crossing, but possibly the

a. Proc. Paper 1331, August, 1957, by Walter C. Carey and M. Dean Keller.
1. Asst. to Chf. of Eng. Div., New Orleans Dist., Corps of Engrs., U. S. Dept. of The Army, New Orleans, La.
2. Structural Engr., Bedell & Nelson, Cons. Engrs., New Orleans, La.

nature of the material on the bed surface at a given moment. With respect to Mr. Lane's thoughts as to the influence of the sea on slopes in the reach under discussion, the writers' observations still further up the river revealed similar phenomena. Furthermore, the systems of waves below Baton Rouge, where the river runs through a deep trough (sometimes in the Pleistocene) did not apparently differ from those above Baton Rouge, where the river is pretty much the same for 600 or 700 miles.

The writers were pleased to note that all of the discussions point to the need for more study of systematic river bed variations with respect to their possible effects on water surface slopes and flow conditions.

An added point, the writers wish to report a recent technological advance that makes it easier to profitably study and compare Fathometer rolls, for such studies as those under discussion: Ordinarily the Fathometer roll is most inconvenient to handle and, being opaque, cannot be compared conveniently with another roll, even when to the same scale. A method has been developed for continuously reproducing Fathometer rolls on translucent vellum by the so-called "electrostatic" process. The reproduced rolls can readily be superimposed on one another, or traced, or used for making reproductions.

CHARACTERISTICS OF FLOW OVER TERMINAL WEIRS AND SILLS[a]

Discussion by F. Paderi
Closure by P. K. Kandaswamy and Hunter Rouse

F. PADERI.[1]—The writer has read with pleasure this paper which demonstrates such explicit recognition of the persistent theoretical and practical nterest in the subject of Basin weirs. He also appreciates the comprehensive portrayal of the problem for the entire range

$$0 \leq \frac{h}{h+w} \leq 1 \tag{1}$$

ind he finds of particular merit the care and breadth of view with which the uthors revise and refine their own prior results to approximate more closey the true phenomenon.

Along these same lines, in many of his own works from as early as 1937,[1,2] the writer has studied this important phase of hydraulics both inalytically and experimentally. These works are also to be considered as steps toward a progressive deepening of knowledge on the subject.

One such item of accomplishment was contained in the paper[3] presented o the Sixth Congress of the International Association for Hydraulic Research it The Hague in September 1955. This paper placed the fundamental contributions of Böss, Rouse and Ippen[4] in evidence and indicated the conservative nature of the limitations, for practical purposes, of the writer's nonlimensional expression for the discharge coefficient

$$\frac{2}{3}C_d = 0.707 - 0.302\sqrt{1-\left(\frac{h}{h+w}\right)^2} \tag{2}$$

n the relationship

$$q = \frac{2}{3}C_d\sqrt{2g}\,h^{3/2}$$

or the calculation of the capacity of overflow dams over the whole of range (1).

. Proc. Paper 1345, August, 1957, by P. K. Kandaswamy and Hunter Rouse.
. Ass't. Prof. of Hydraulics, Univ. of Pisa, Pisa, Italy.

As was pointed out in the foregoing paper, the use of Eq. (2) in the case of overflow dams of the proper profile is justified by experiments on Basin weirs which confirm the existence of such a trend.

The numerous experiments of Meyer and See[5,6] and those more recently performed in the hydraulics laboratory of the U. S. Bureau of Reclamation[7] should also be cited.

In his studies on the Basin type of weir, the writer started with a determination of the merits and limits of validity—as recognized by the authors—of the empirical expression of Rehbock for C_d and of the theoretical treatment of von Mises,[8,9] and he warned about the controlling effect of the phenomena of the second critical stage.

The writer dealt, in effect, with a homogeneous perfect fluid starting from rest under the sole action of gravity (and hence in irrotational motion). He applied the theorem of Bernoulli and also—after Boussinesq,[10,11] who found the constant value $2C_d/3 = 0.4342$, and De Marchi[12]—the theorem of momentum, and arrived at an analytic expression for the discharge coefficient C_d of the Basin type of weir.

Following the inverse procedure, he then began with the theoretical and experimental condition of the second critical stage ($C_d = 3/2\ \sqrt{2} = 3 \times 0.707/2 = 1.06$) and, without experimental aid, determined the value $C_d = 3 \times 0.4048/2 = 0.607$ at the other extreme $h/(h + w) = 0$ of range (1), as was then expressed by Eq. (2). The distribution of velocity at the section of measurement was assumed practically uniform.

The writer does not consider it worthwhile to dwell upon details of the subject; however, he does believe it to be of general interest to note that, starting with the study of 1937, these results were obtained by a procedure analogous to that which he has presented in his discussion of Proceedings Paper 1038.[13] This procedure consists of the introduction, at a certain point in the analytical treatment, of a coefficient K (of the class K_3), which resulted analytically in a value equal to 2 for the theoretical and experimental conditions of the second critical stage.

The last word still remains to be said regarding the experimental aspect of this problem; the writer—after Rouse—has long concerned himself with this aspect of the free overfall for both horizontal and sloping channels.[13,14]

Laboratory practice warns against experiments at too small heads, because of related phenomena which complicate the laws of similitude. Perhaps certain phenomena are also brought into existence by small heights of weir. These phenomena could possibly be connected with the corresponding distribution of velocity.

One particular phenomenon has been perceived by the writer in the course of his long experimentation on the free overfall: the accentuation of the last undulation just before the final drop in surface profile as subcritical flow approaches the brink (channel of mild slope). This is related to local disturbances of the flow at the inlet of the test channel. Consequently, the theoretical critical section of depth

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

is shifted slightly away from the brink, and hence the absolute value of x/h becomes greater than x_c/y_c.

The writer observes, however, that the second critical stage has a par-
cular constant physical characteristic of stability. This limits the conse-
uences of such disturbances. It therefore becomes feasible, because of the
mited slope of the free surface in the critical region, to use this character-
stic to realize an "overfall-brink discharge meter." The corresponding
xperiments are in progress.(15)

That all these fields of research offer great possibilities of development
s attested by the authors' indications, which are as valuable from the ex-
erimental side as for their theoretical repercussions.

P. K. KANDASWAMY,[1] A.M. ASCE and HUNTER ROUSE,[2] M. ASCE.—Dis-
ussions of the writers' presentation by Messrs. Engel and Paderi are much
ppreciated for the sidelights that they throw upon the presently incomplete
nderstanding of the phenomenon under consideration. Mr. Engel states that
ie writers have not explained when and why it occurs; and Mr. Paderi, in his
wn work on the problem, finds rather different results. Hence the question
aised seems to involve not only when and why but also whether.

Mr. Engel prefers the earlier empirical idea of a base coefficient (which
e believes to decrease as the head-depth ratio increases) and an apparently
ndependent velocity-of-approach coefficient. As explained in the paper, both
ie contraction coefficient and the velocity-of-approach coefficient (which to-
ether determine the discharge coefficient) are functions of, and increase
rith, the head-depth ratio throughout the weir range. Mr. Paderi, proceeding
n the latter basis, has derived a discharge relationship that increases con-
nuously to the free-overfall limit. Although he has no experimental data to
onfirm this relationship in the vicinity of its upper limit, several series of
ata in the lower range duplicate its trend. As Mr. Paderi has pointed out,
owever, other series of data exist which duplicate the trend of the writers'
ndings.

If one approaches the zone in question from the other direction, it must be
oted that the original observation by Böss—that a very low sill can be placed
t the brink of a free overfall without appreciably increasing the depth of the
pproaching flow—is readily verified. It follows that the discharge coefficient
iust attain a maximum value at some intermediate sill height. Evidence of
uch a reversal in trend can also be seen in the basic difference in nappe-
rofile variation for the weir and sill ranges (compare Figs. 4 and 5).
Whether the change in function occurs and when it occurs thus seem to be
nswered satisfactorily by the writers' diagrams. Why it occurs is probably
ubject to the same sort of answer as why a sluice gate has the effect that it
oes as it begins to penetrate the surface of flowing water; such gravitational
henomena are more readily subject to analytical evaluation (for example, by
elaxation) than to descriptive explanation.

Mr. Paderi has pointed out still another aspect of wave formation in its
elation to the profile measurements, and both he and Mr. Engel have men-
oned the possible effect of surface resistance. Each of these factors un-
oubtedly has bearing upon the discrepancies that are evident not only between
ie independent measurements of the two writers but to a much greater extent
mong sets of data from wholly unrelated investigations. As the writers

. Iowa Inst. of Hydr. Research, State Univ. of Iowa.
. Prof., Fluid Mechanics & Div., Inst. of Hydr. Research, State Univ. of
Iowa.

showed, the two influences can be minimized—but never wholly eliminated—by using as large a radius of floor curvature and as short a subsequent region of uniformity as possible, and by measuring the head consistently only sufficiently far upstream to avoid the final zone of curvilinearity.

Mr. Engel appears to disregard the effect of curvilinearity upon the critical-depth relationships developed for parallel flow, since he uses these to relate the depth over the sharp-crested weir to that over the broad-crested weir and to the head upstream. He also raises questions as to the magnitudes of the actual depth ratios, which—if the writers understand his query—can all be read directly from the several diagrams that they provided. The critical depth for curvilinear flow is a matter that still remains open to generalization; the writers will hence be interested in following Mr. Paderi's development of an overfall-brink discharge meter.

TURBULENCE IN A DIFFUSER BOUNDARY LAYER[a]

Closure by J. M. Robertson and G. L. Calehuff

J. M. ROBERTSON,[1] M. ASCE and G. L. CALEHUFF.[2]—This paper was ntentionally kept reasonably short and as uncomplicated as possible in the iope that a larger number of people could follow and understand it. The basic relations were therefore presented in the simplest fashion. As Mr. Strelkoff points out, this resulted in some inexactitude of statement in regard to the urbulence production terms. The expression calculated for P_r represents only part of the energy transformed from the mean flow into turbulence. The complete turbulence production relation is the product of Reynolds stresses and mean velocity gradients. In tensor notation this may be written as

$$P_r = \overline{u_i u_j} \; \frac{\partial U_i}{\partial x_j}$$

For plane two-dimensional flows the nine terms implied by the above expression reduce to the following four:

$$P_r = u'^2 \frac{\partial U}{\partial x} + v'^2 \frac{\partial V}{\partial y} + \overline{uv} \; \left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)$$

This is Equation 4 of Schubauer and Klebanoff's(2) report. In their evaluation of the several production terms, the third term was found to contribute roughly 80 percent of the total while the fourth term was negligible. From this indication the P_r values presented in Fig. 9 should not be in error by more than 20 percent.

The flow situation reported on in the present paper did not represent a simple two-dimensional system, since the boundary layer developed along a onical diffuser wall. However in view of the geometry[3] of the experimental rrangement, it would seem that the above simplified production relation in erms of an x-y coordinate system located at the wall is approximately correct. On this basis the four terms have been calculated and are presented in Fig. 10. The values of $\partial U/\partial x$ utilized to calculate the first term in the P_r xpression were found by numerical differentiation of the pitot tube data(9,10) etween station 12 and stations 5 inches upstream and 2.5 inches downstream.

. Proc. Paper 1393, October, 1957, by J. M. Robertson and G. L. Calehuff.
. Prof. Theo. and Appl. Mchs., Univ. of Illinois, Urbana, Illinois.
. Project Leader, Hydraulics Group, West Virginia Pulp and Paper Co., Covington, Virginia.
. The virtual apex of the conical surface was 50.7 inches upstream of station 12.

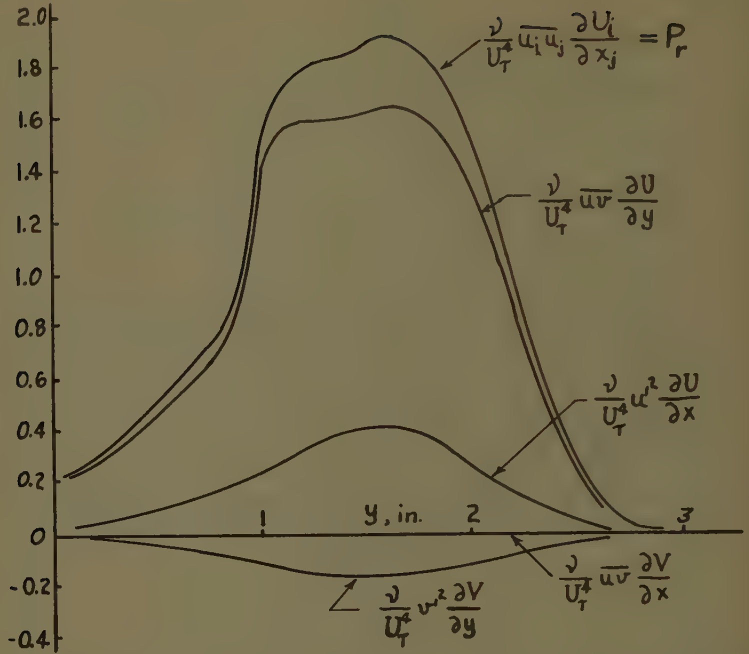


FIG. 10 VARIATION IN TURBULENCE PRODUCTION TERMS ACROSS BOUNDARY LAYER AT STATION 12

The second term was similarly evaluated with the aid of the continuity relation $\frac{\partial V}{\partial y} = -\frac{\partial U}{\partial x}$. The last term involves $\partial V/\partial x$ which was estimated from a transverse integration of the second difference of U with respect to x. The result was very approximate, but as in the case analyzed by Schubauer and Klebanoff, this turbulence production term was negligible, being the order of one-half a percent of the total at any point. The general variation of the four turbulence production terms is quite similar to that found by Schubauer and Klebanoff. The major term in this case is nearer 90 percent of the total.

A complete presentation of the "history" theory for the shear stress was not included since this is adequately covered in the literature.(13) Mr. Strelkoff's discussion of the relation between the lateral shear stress gradient and the longitudinal pressure gradient is in agreement with the theory. As noted in the paper, this relation applies near the wall and is so included in the theory. In the outer region the "history" effect enters and the lateral shear stress gradient is assumed to be that occurring at the start of the pressure gradient. All of the terms in Fig. 6 are defined in the paper except $\tau_{w_{o_e}}$.

This represents an effective (hence the last subscript e) wall shear at the initial station. It was taken as 1.2 times the actual value of the shear in order

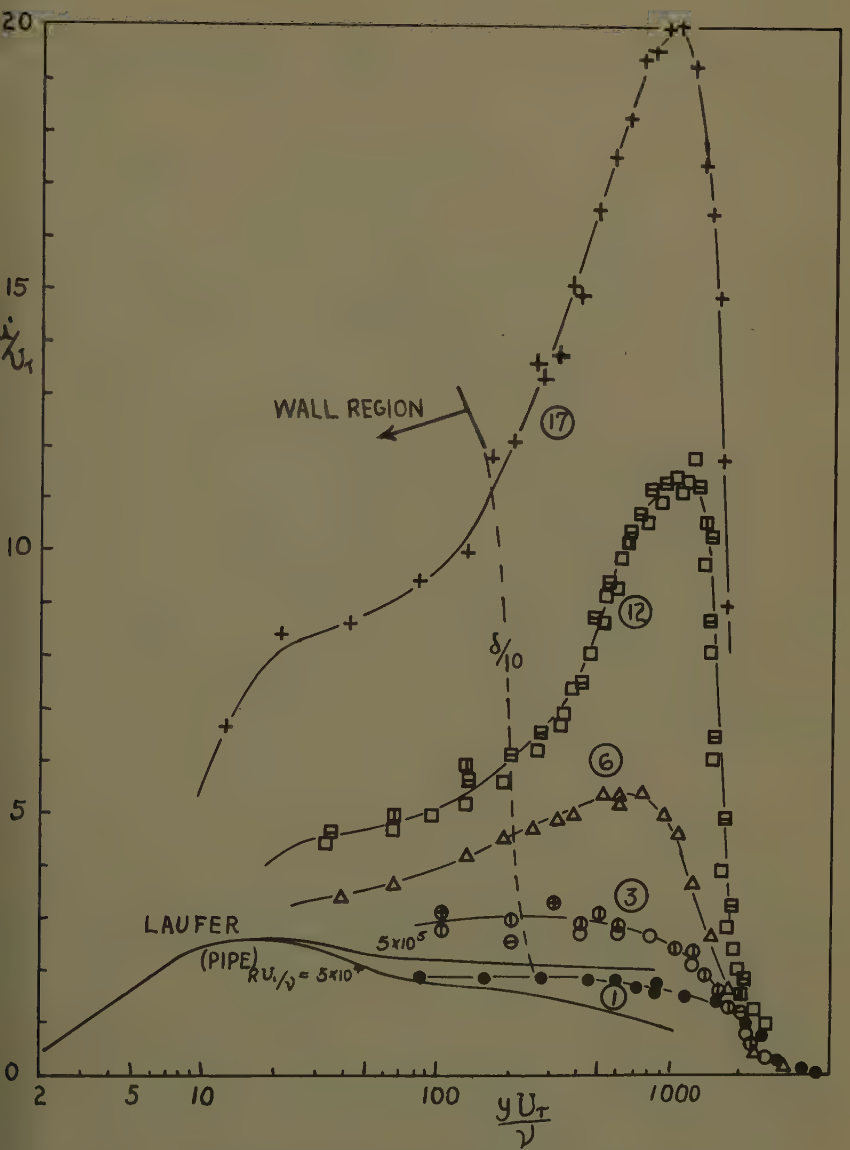


FIG. 11 TURBULENCE INTENSITIES AT VARIOUS STATIONS IN TERMS OF SHEAR VELOCITY

to define the initial slope of the shear stress in the outer portion of the boundary layer. It is this slope which the history theory assumes invarient as the boundary layer progresses into the pressure gradient.

The turbulence intensities can be presented as any of four dimensionless ratios; three of these were employed in the paper. Mr. Strelkoff suggests that if we had utilized the fourth u'/U_1 that the peak would have appeared at each station. This peak is clearly evident in the contour plot of Fig. 5 and is seen to drift to a location near the middle of the boundary layer as the layer develops. Nothing new in this regard would result from a presentation of u'/U_1 instead of u'/U_{10}. The ratio u'/U_τ was utilized (Fig. 8) only for the data at station 12. As Mr. Strelkoff suggests, such a presentation of the data at all stations is quite informative, as shown in Fig. 11. A progressive deviation from the pipe results of Laufer is apparent even in the wall region (inner tenth of the boundary layer). The measurements for the first station seem in good agreement with the pipe data. The turbulence levels in the outer boundary layer region appear very high indeed since they bear no relation to the local wall shear stress and hence U_τ.

The zero, or wall reading, of the hot wire was obtained by adjusting the traverser until the wire was a small, but known distance from the surface, as noted with a scale with a least reading of 0.05 inches. No corrections to the hot wire readings were applied due to wall proximity. The heat loss to the wall probably had some effect on the mean velocity readings. Effects on turbulence indications are possible but the authors are not aware of any information on this subject. Laufer's measurements were in a brass pipe and a wooden channel while Klebanoff's boundary layer was on an aluminum plate.

The comments by Mr. Strelkoff are appreciated. The authors are in complete agreement with him as to the desirability of further work along this line. The flow in a similar ten degree diffuser is under study at the University of Illinois but no hot wire anemometer measurements have been attempted in it yet.

REFERENCES

2. "Investigation of the Separation of the Turbulent Boundary Layer," by G. B. Schubauer and P. S. Klebanoff, NACA Report 1030, 1951.

9. "The Growth of an Axisymmetric Turbulent Boundary Layer in an Adverse Pressure Gradient" by E. M. Uram, Proc. 2nd U. S. National Congress of Applied Mechanics 1954, ASME, pp. 687-695.

10. "Effect of Adverse Pressure Gradients on Turbulent Boundary Layers in Axisymmetric Conduits" by J. M. Robertson and J. W. Holl, Journal of Applied Mechanics, June 1957, pp. 191-196.

13. "Shear Stress in a Turbulent Boundary Layer" by D. Ross and J. M. Robertson, Journal of Applied Physics, vol. 21, 1950, pp. 557-561.

100 FREQUENCY CURVES OF NORTH AMERICAN RIVERS[a]

Closure by E. Kuiper

E. KUIPER,[1] M. ASCE.—The author appreciates very much the discussion f his paper by Messrs. Riggs, Alexander, Kohler, and Powell, and believes at the points they have brought out have added materially to its value. The thor was particularly gratified that the main thesis of the paper "that freiency curves of maximum annual flood peaks can be drawn with greater ccuracy when due attention is paid to other frequency curves on the same or mparable streams" was found acceptable.

Mr. Riggs is to be thanked for his analysis of 17 stations, deriving their incipal floods from rainfall, and pointing out that a very close relationship ay exist between the one per cent flood as an independent variable and the ean flood and average discharge as dependent variables. This would seem indicate that drainage basin characteristics, including the prevailing cliate, are sufficiently expressed in the mean flood and average discharge, d do not have to enter the relationship as separate parameters. This would mplify the problem considerably. The author hopes to find time in the near ture to pursue this thought and test it on other streams, possibly with differnt functional relationships.

Mr. Alexander has attempted to make a theoretical analysis of the thesis hat an increase in the size of the drainage area tends to flatten the freiency line" and finds that actual analysis of existing frequency curves is quired. He further states that "such analyses now being undertaken by the . S. Geological Survey do support this thesis. The author and some of his ssociates, at one time, made a similar attempt to provide theoretical proof, combining assumed frequency curves by graphical procedure, and introcing correlation and coincidence of flood peaks. The conclusion of this alysis was that frequency curves do flatten indeed with an increase in drainge area. This analysis was not included in the paper since it is rather inlved, and since there is ample and conclusive proof in the data presented in able 7.

Mr. Kohler made a very valuable contribution to the paper by presenting s chart, relating the one per cent flood, the mean annual flood and the size the drainage area of the 100 river stations under discussion. The author as particularly impressed by the close relationship of the mean annual flood d the one per cent flood, regardless of the size of the drainage basin. For proximate estimates of the one per cent flood in cases where the mean anal flood is known, the line shown in the chart will serve a very useful purse. In cases where the mean annual flood is not known, the lower part of

Proc. Paper 1395, October 1957, by E. Kuiper.

Chief Engr., Manitoba Water Resources Investigation, Manitoba, Canada.

the chart would provide guidance. One could select a line representing river stations with similar drainage basin characteristics, enter the diagram with the given size of the drainage area, find the mean annual flood and the one per cent flood. Such a chart would undoubtedly gain in value when more information is added.

Mr. Powell is quite right in pointing out that the median annual flood is less susceptible to an unbalanced sample than the mean annual flood. However, most of the work on frequency curves that has been done in the past has been in terms of the mean annual flood. In order to facilitate comparison of past and present data, the author has adhered to this parameter. Mr. Powell suggests that a formula may be developed for estimating the one per cent flood directly. This suggestion could very well materialize if one pursued the valuable contribution of Messrs. Riggs and Kohler and tested them against available data in various drainage basins.

THE HYDRAULIC DESIGN OF STILLING BASINS: HYDRAULIC JUMPS ON A HORIZONTAL APRON (BASIN I)[a]

Closure by J. N. Bradley and A. J. Peterka

J. N. BRADLEY[1] and A. J. PETERKA[2] —In discussing the length of jump as measured by the writers, Mr. Thomas and Mr. Elevatorski raise several questions. Mr. Thomas feels that the writers used an arbitrary point for jump length determination and that the point used in each test was not specified, i.e., whether the point on the roller or the point where the jet began to leave the floor was the determining factor. In the section "Forms of the Hydraulic Jump," the determining factor is discussed in terms of the Froude number ranges illustrated in Figure 9. Thus, for any particular test the determining factor for the length of jump criterion is reflected in F_1, Column 10, Table 1.

Mr. Thomas feels that it would have been preferable to record the length of roller as the length of jump in all cases. Mr. Elevatorski voices somewhat the same opinion when he states that he prefers the definition" * * * the point where no return flow is visible." Had these points been used, the jump lengths for Froude numbers less than 4.5 would have been too short since for Froude numbers up to 4.5 the point at which the high velocity jet begins to leave the floor is the controlling factor and the roller is relatively short. Serious downstream erosion might result if a stilling basin were constructed only as long as the roller. The bed was smooth at all times so the measurements could not have been affected by the roughness of the bed as Mr. Thomas assumes. The writers believe that the length determinations given in the paper are not based on arbitrary conditions, as stated by Mr. Thomas, but rather provide a more realistic evaluation of the length of the jump than has heretofore been presented. The writers devoted several days to observing jumps of various sizes and Froude numbers before formulating criteria for determining the length. The criterion suggested by Messrs. Thomas and Elevatorski applies to only one type of jump. The writers wish to emphasize again that the length of jump was chosen from a practical viewpoint; the end of the jump represents the end of the concrete floor and training walls of the shortest stilling basin possible. In view of the practical nature of the tests, it was not considered important that the jump length was not" * * * easily observed, and a characteristic parameter of the jump * * *" as stated by Mr. Thomas. It was considered important, however, that the jump length be

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a. Proc. Paper 1401, October, 1957, by J. N. Bradley and A. J. Peterka.

1. Hydr. Engr., U. S. Bur. of Public Roads, Washington, D. C., formerly of U. S. Bureau of Reclamation, Denver, Colo.
2. Hydr. Engr., U. S. Bureau of Reclamation, Denver, Colo.

chosen to represent the shortest stilling basin possible, consistent with good performance and a reasonable factor of safety.

The experiences of Mr. Thomas with canal falls in India are interesting and it is possible that rapid side expansions in the stilling basin were the cause of lateral oscillations resulting in poor performance. In the writers' experience, wall angles of 6° to 8° provide safe divergence. Larger angles may produce unstable conditions caused by a contraction of the flow width in the basin rather than an expansion. However, baffle piers help to increase the allowable angle of divergence but no data are available as to the degree.

Mr. Thomas states that insufficient evidence is presented to conclude that baffle piers are of little value in a stilling basin when F_1 is between 2.5 and 4.5. Paper 1404 covers this problem thoroughly and Figure 21 shows a few of the many arrangements which were tested in an attempt to use baffle piers. In this low Froude number range, conventional baffle piers do more harm than good, and they cannot be used to stabilize the flow or reduce the length of basin required. Conventional baffle piers were found to create instability and it was necessary to lengthen the basin to obtain flow conditions equivalent to those in a basin without them. The only effective arrangement found by the writers is shown in Figure 22. Flow currents passing over the tops of the oversized chute blocks tend to reinforce the roller to provide a more stable jump.

Mr. Elevatorski has indicated other methods of expressing the writers' data, but the writers do not see an advantage in altering the method of presentation. Before a designer completes the design of a stilling basin, he will have need for the value of V_1, and calculation of the Froude number is as easy as the calculation of the critical depth. The Froude number is used several times in the design of a basin using the writers' methods. The writers also prefer expressing the jump length in terms of D_2 rather than in terms of $D_2 - D_1$ as suggested by Mr. Elevatorski and Mr. Thomas, because in practice it has been found to be convenient. The writers were aware of the relations described by Mr. Elevatorski since they were first worked out by Riegel and Beebe in 1917.*

Mr. Elevatorski's concern over the lower part of the curve in Figure 5 is not understandable by the writers since, obviously, there is very little choice in the manner in which the curve may be extended from $F_1 = 2$ (the last test point) to $F_1 = 0$. Further, there is no mention or inference that an undular-type jump will be formed at Froude numbers less than 2. On the contrary, Figure 9 shows that the prejump type of action will occur. The writers should have stated, however, that the portion of the curve from $F_1 = 1$ to $F_1 = 0$ has no particular significance.

The writers concur with Mr. Elevatorski's comment that the submitted information should not be used to design trapezoidal stilling basins, unless, of course, the increased area introduced by placing a batter on the side walls is a very small percentage of the total cross-sectional area.

* The Hydraulic Jump as a Means of Dissipating Energy, Technical Reports Part III, the Miami Conservancy District State of Ohio, Ross M. Riegel and John C. Beebe.

Corrections to Proc. Paper 1401

Page	Correction
1	Change forms to form in line 5
7	Line 4 of the Introduction—Change Matske to Matzke
23	Item 4 should read "No particular difficulty is encountered in the form shown in Figure 9c. Arrangements of baffles and sills will be found valuable as a means of shortening the basin. This is discussed in Papers 1402 and 1403.

HYDRAULIC DESIGN OF STILLING BASINS: HIGH DAMS, EARTH DAMS AND LARGE CANAL STRUCTURES (BASIN II)[a]

Closure by J. N. Bradley and A. J. Peterka

J. N. BRADLEY[1] and A. J. PETERKA.[2]—The fact that the titles of Papers 1402 and 1403 contain suggested uses of the basin should not influence the designer in his choice of basins. The only reason for using a Type II basin rather than a Type III basin is the possibility of cavitation damage resulting from the use of baffle piers.

As Mr. Thomas suggests, there is no doubt that a row of baffle piers is a more effective energy dissipator than a row of chute blocks. However, if baffle piers are to be used there is no advantage in installing them in a Type II basin. Rather, a Type III basin should be used and the resultant savings from reducing the basin length and simplifying the end sill can be realized. In Paper 1403, Introduction and Recommendations, the limiting discharge and velocity for use of a Type III basin are given.

The length of basin in terms of scour downstream can be clarified in the original paper by pointing out that Table 2 (which was printed in Paper 1403 but which should have been printed in Paper 1402) lists some 36 basins that were model tested using a sand bed downstream in which extensive erosion tests were made. Since the recommended Type II basin is patterned after these, and since some of the verification test models contained a sand bed, it is known, but was not stated in the writings, that the recommended structures will be satisfactory for installation where the scour problem is of average difficulty. Figure 12 shows that the majority of existing basins similar to Type II are shorter than the recommended Type II basins. Since the scour patterns were satisfactory for the short basins, the scour patterns for the longer basins will also be satisfactory.

The drawing in Figure 10, criticized by Mr. Thomas, is not a recommended design. Rather, it is, as the drawing title states, a "Definition of Symbols" and was intended to be used with Table 2. Some of the early stilling basins had the end sill upstream from the end of the apron. This is not now considered good practice. Figure 14 shows the recommended proportions of the Type II basin.

The direct solution proposed by Mr. Thomas in Figure 1 for determining apron elevation may be used satisfactorily. However, the trial and error difficulties mentioned by Mr. Thomas in connection with the use of Figure 11 will be found to be minor in nature when solving practical problems. The change

a. Proc. Paper 1402, October, 1957, by J. N. Bradley and A. J. Peterka.
1. Hydr. Engr., U. S. Bureau of Public Roads, Washington, D. C., formerly of U. S. Bureau of Reclamation, Denver, Colo.
2. Hydr. Engr., U. S. Bureau of Reclamation, Denver, Colo.

in D_1, V_1, and F_1 occasioned by changing the estimated elevation of the apron over the usual possible range will be found to be relatively small. A single correction is sometimes necessary, but this is usually done only to be certain that the first assumption was correct. For example, if a spillway discharges 14 cfs per foot with a 2.5-foot head on the crest over a fall 50 feet high, $F_1 = 14.13$. If the apron elevation was assumed to be 5 feet too high and it is necessary to recalculate for 55 feet of fall, $F_1 = 14.28$. $\frac{TW}{D_1}$ would then change from 19.4 to 19.8 and D_2 would change from 6.05 feet to 6.12 feet. The overall error in not recalculating after repositioning the apron would be less than 1 inch. In the same problem, if the discharge per foot of width is increased to 41 second-feet the error would be less than 2 inches. Both errors are negligible.

Corrections to Proc. Paper 1402

Page	Correction
4	Under Verification Tests there is a reference to Table 2. Table 2 was printed, however, in Paper 1403. Table 2 should be printed adjacent to this first reference.
7	In the section Aids in Computation there is a reference to Figure 15. Figure 15 should be printed here rather than in Paper 1403.

HYDRAULIC DESIGN OF STILLING BASINS: SHORT STILLING BASIN FOR CANAL STRUCTURES, SMALL OUTLET WORKS, AND SMALL SPILLWAYS (BASIN III)[a]

Closure by J. N. Bradley and A. J. Peterka

J. N. BRADLEY[1] and A. J. PETERKA.[2]—The discussion by Mr. Thomas follows two main trends. In one he has preferred to express the writers' variables in other terms, perhaps more familiar to him. This is a justifiable procedure and the choice is up to the individual. In the other he has interpreted certain criteria contrary to the meaning and use intended by writers. In these, the writers will explain the original concepts further to insure that other readers will understand the true significance of the data and analysis presented.

Mr. Thomas feels that the use of his Figure 1 simplifies the vertical placement of the apron by eliminating the cut and try process. As demonstrated in a sample problem in the writers' closing discussion of Paper 1402, the writers believe that in most installations the errors involved in a first reasonable assumption are small and may be eliminated with one relatively simple correction. They prefer this method, when using the overall procedures given in the paper, to the one explained by Mr. Thomas which requires the calculation of additional values not otherwise used.

The necessary depth, D_2, for a Type III basin is given in Figure 11 and this is shown to be 15 to 18 percent less than the conjugate depth computed for a hydraulic jump, depending on the Froude number. In the paragraphs under the heading "Tail-water Depth" it is recommended that full conjugate depth be used for a Type III basin. The reasons are stated. Mr. Thomas states in Item 1 of his discussion that the basin floor may be raised because of the greater factor of safety against jump sweepout in a Type III basin. If only the arithmetic in the problem is considered, the basin floor may truly be raised. However, in the light of past experience with prototype structures where operating difficulties are often experienced because of a lack of tail-water depth and rarely, if ever, because of too much depth, the writers decided to recommend full conjugate tail water. The basin then contains a factor of safety against phenomena of which the designer is often unaware such as retrogression of the stream bed or a poorly derived tail-water curve. In addition, full conjugate depth will provide improved performance including general appearance, velocity distribution, wave and surge formation, and scour. Safe, but minimum, performance will result if the designer chooses to use the minimum tail-water value shown in Figure 11.

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a. Proc. Paper 1403, October, 1957, by J. N. Bradley and A. J. Peterka.
1. Hydr. Engr. U. S. Bureau of Public Roads, Washington, D. C., formerly of U. S. Bureau of Reclamation, Denver, Colo.
2. Hydr. Engr., U. S. Bureau of Reclamation, Denver, Colo.

Mr. Thomas states that the length of jump is more directly a function of $D_j = D_2 - D_1$, than of D_2. The writers chose D_2 because they found it more convenient to work with one variable rather than two. Either method is acceptable however.

Mr. Thomas states that the length of basin required is not necessarily the same as the length of jump as measured by the writers. The basin length measured by the writers, as stated in Paper 1401, is as follows:

> "It was the intention to judge the length of the jump from a practical standpoint; in other words, the end of the jump as chosen would represent the end of the concrete floor and walls of a conventional stilling basin."

The jump length in Basin I and the basin lengths for Type II and Type III were all judged on the same basis. For similar flow conditions, Basin I, Basin II, and Basin III will provide the same degree of protection against scour and in many cases will provide nearly identical overall performance based on the resulting downstream flow conditions. Representative erosion tests were made (but not reported) in sufficient numbers to provide assurance that the recommended basin lengths were sufficient to prevent undermining of the end sill and excessive scour downstream. In fact, scour was one reason for not reducing the basin lengths still further. Complete scour test data were not recorded because of the difficulties in obtaining and presenting meaningful erosion data. The writers are confident that the scour problems which are encountered at the ends of training walls and other areas which cannot be evaluated in a sectional model will be of greater concern to a designer than the scour encountered downstream from the recommended stilling basins. On the other hand, the writers believe, and have advised in several places in the group of papers, that model tests should be performed to check the recommended designs whenever unusual conditions are encountered on a particular structure. Certainly, an easily eroded downstream channel bed should be included in this category.

The position of the baffle piers on the apron is given by the writers as $0.8\ D_2$. It was found that if the piers were placed closer to the toe of the jump, the individual jets of water rising vertically from each pier penetrated the covering tail water resulting in incomplete stilling action. As the piers were moved downstream from the $0.8\ D_2$ position, the usefulness of the piers in creating turbulence decreased rapidly. If the basin entrance velocities are sufficiently high, cavitation at the baffle piers is most likely to occur when the piers are in the upstream position. However, data taken on one set of piers since Paper 1403 was written indicate that moving the piers downstream does not reduce the possibility of cavitation as much as might be expected. For $F_1 = 10$, $q = 240$ cfs, and $V_1 = 100$ feet per second, the lowest pressure measured on a sharp-edged baffle pier placed $0.64\ D_2$ downstream from the start of the jump was -20 feet of water. When the pier was moved to $0.98\ D_2$, the pressure increased but only to -10 feet of water. In the downstream position, the effectiveness of the piers was reduced and the basin did not perform as well as a standard Type III basin.

Moving the baffle piers farther downstream resulted in greater loss of performance. Streamlining the piers and placing them in the upstream position resulted in considerable loss in effectiveness. In fact, when the piers were streamlined sufficiently to produce acceptable pressures, the turbulence-creating qualities were so reduced that it was doubtful whether the use of the streamlined piers could be justified. The piers were less effective than

ndicated by the area of the vertical face. Because of the many possibilities present as illustrated above, no flexible arrangement of baffle piers could be given in the original paper.

The limiting factors concerning the use of baffle piers placed 0.8 D_2 downstream from the toe of the jump are given in the paper as V_1 = 50 feet per second and q = 200 cfs per foot of basin width. These values contain a judgment factor by the writers, but until more data are available the stated limits should be used and the baffle piers should be placed at 0.8 D_2 from the chute blocks, regardless of the absolute size of structure. Mr. Thomas suggests, "* * * in a large work, blocks must be placed at a greater relative distance from the toe than in smaller works with equal F_1." This is not so. If hydraulic models can be accepted as a means for determining stilling basin dimensions, the absolute size of structure has nothing to do with the position of baffle piers relative to the toe of the jump. The writers' tests apply to large structures as well as small ones.

Mr. Thomas has failed to realize the significance of Figure 19. The profile represents the approximate water surface profile for a Type III basin operating at conjugate depth. It does not show, as Mr. Thomas states, "* * * the tail water had been lowered to the stage when the beginning of the jump had receded from the toe and reached the blocks * * *"

Mr. Thomas feels that the writers dismissed the possibility of using a second row of baffle piers without due consideration. As stated in the paper, tests with two rows of baffle piers were made for each arrangement of baffle piers tested. Although the second row of piers did improve the performance, the improvement was not sufficient to allow reducing the basin length further.

There is no doubt that in the writers' tests the chute blocks created turbulence in the upstream portion of the basin, helping to make the second row of baffle piers relatively ineffective. The writers also believe that where an erosion problem is particularly difficult, a second row of piers might help in reducing scour. It would then be necessary, however, to lengthen the basin by the distance between the rows of piers to maintain the proper distance between the piers and the end sill. For the general case, however, in the judgment of the writers a second row of baffle piers in Basin III cannot be justified.

Mr. Thomas expresses a desire for more supporting data in order that the designer may judge the effects of deviations from the standard Basin III. Since this is a new design and had not been verified by prototype experiences, the design was kept rather rigid. However, studies are being continued to determine the pressures on the baffle piers for variations in the 0.8D_2 dimension, the effect of baffle pier streamlining on the baffle pier pressures, and other features of the Type III basin presently covered in a general way. However, the data presented are sufficient to provide a first design which will provide satisfactory performance with a minimum structure. Refinements or further reductions in dimensions should be accomplished with hydraulic model tests.

Corrections to Proc. Paper 1403

Page	Correction
5	Figure 18 in the last line should be changed to Figure 17.
12 and 13	Table 3 should be printed in Paper 1402.
16	In the last sentence on Baffle Piers, chambers should be changed to chamfers.

HYDRAULIC DESIGN OF STILLING BASINS: STILLING BASIN WITH SLOPING APRON (BASIN V)[a]

Corrections to discussion by A. Rylands Thomas,[1] M. ASCE

CORRECTIONS—The accompanying figure was inadvertently omitted from Mr. Thomas' discussion of Proc. Paper 1405 in Proc. Paper 1616 (April, 1958):

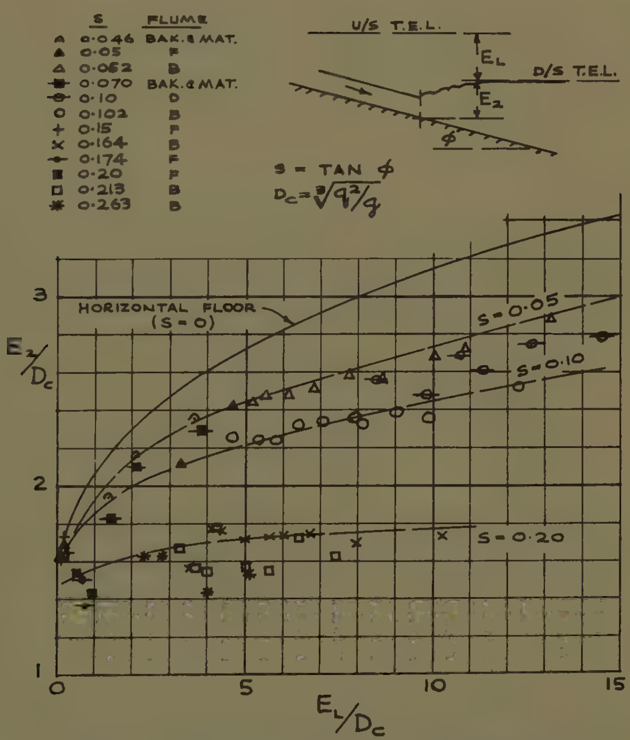

FIG. A. HYDRAULIC JUMP ON HORIZONTAL AND SLOPING FLOORS (CASE D)

a. Proc. Paper 1405, October, 1957, by J. N. Bradley and A. J. Peterka.
1. Consultant, London, England.

THE HYDRAULIC DESIGN OF STILLING BASINS[a]

Closure by J. N. Bradley and A. J. Peterka

J. N. BRADLEY[1] and A. J. PETERKA.[2]—Mr. Nimmo describes Sommer-et Dam which has a relatively complicated discharge system. The data of asin II, therefore, should not be expected to apply in the sense that a basin esigned from the Basin II rules will operate properly with nonuniform flow istribution. On the other hand, if model tests were used to provide sills, tc., to distribute the flow so that the overall effect was the same as uniform-/ distributed flow entering the basin, Basin II criteria may then be used to heck the overall size, depth, and other dimensions of the basin. This, Mr. 'immo has done and found that Basin II would have been slightly larger than ıe basin developed from individual model tests. This helps to substantiate ıe authors' statements that there is a factor of safety inherent in the recom-ıended basins, I-VI.

On Tinaroo Falls Dam, the stilling action described by Mr. Nimmo re-ulted from impact of the flowing water on the apron and on the sills. It is ften possible to provide impact-type structures which are smaller than hy-raulic jump basins. However, impact-type structures must be tailored to fit ıe exact flow conditions encountered and judgment and experience are also equired to provide a satisfactory stilling structure.

Mr. Rand feels that more flexibility should have been provided so that the esigner could vary the dimensions of the basin to fit individual cases. An nalysis which would completely evaluate each variable would provide an leal solution but would have required a testing program too involved and ngthy to complete.

To obtain the most usable data from a reasonable expenditure of time and ınds, the authors evaluated the major variables, basin length and depth, in rms of fixed arrangements of blocks, piers, and sills placed to provide the mallest stilling basin consistent with good performance. The arrangements ıosen resulted in the least tail-water depth and/or basin length. Thus, if ıore flexibility in the appurtenances had been provided, a larger stilling asin would have been necessary for each arrangement. The major basin imensions, depth and length, usually determine whether the designer can conomically justify the use of a hydraulic jump stilling basin. If he can, he as only to place the recommended appurtenances in the basin to obtain opti-ıum basin performance. Thus, there is no primary need for alternate baffle ier sizes or locations or other minor variations.

. Proc. Papers 1401 through 1406, October, 1957, by J. N. Bradley and A. J. Peterka.

. Hydr. Engr., U. S. Bureau of Public Roads, Washington, D. C., formerly of U. S. Bureau of Reclamation, Denver, Colo.

Hydr. Engr., U. S. Bureau of Reclamation, Denver, Colo.

As stated in the papers, the dimensions and spacing of the blocks, piers and sills are not critical. Departures from exact dimensions may be made to avoid construction joints, fractional blocks, etc. Thus, a complete analysis of the basin would have served mainly to prove that the blocks, piers and sills were of optimum size and in the best arrangement. As the material is presented, however, it is necessary for the reader to rely on the judgment of the authors to some degree, although the data contained in the tables show the range of the investigation concerning other arrangements of basin appurtenances.

Despite Mr. Rand's objections to the authors' use of the word "generalized," the authors feel that they have generalized the design of several types of stilling basins, since data are presented which make it possible to design a stilling basin for most any combination of discharge and height of fall. To provide generalization to the degree suggested by Mr. Rand would require an almost impossible amount of testing and data preparation and would not provide more usable designs. A more reasonable approach to the problem is to generalize the design of other types of basins which experience shows to be in demand. When more of these have been completed, the designer will have a greater variety of basins from which to choose.

Mr. Rand poses several questions concerning the jump length and the basin length as measured by the authors. These can probably best be answered by reviewing the basic objectives of the test program. In Basin I, the length of paved apron is the same as the length of the jump. In Basin II, chute blocks were added to the basin to promote stability in the jump and to increase the factor of safety against jump sweepout at tail-water depths less than conjugate. The end sill was then moved upstream on the paved floor until the flow conditions downstream from the dentated sill were just beginning to become objectionable with regard to scour and water surface roughness downstream. In this position, the high point of the boil was at or upstream from the end sill; in other words, the water surface did not continue to rise beyond the end of the basin training walls to allow inflow into the basin from the true tail-water surface behind the training walls. The jump length was found to be less for Basin II than Basin I, although the action at the ends of both basins is similar. The end sill was, therefore, not subjected to a particularly high velocity current and its primary purpose was to lift upward the remaining bottom currents to reduce scour. The jump is therefore "forced" only to a slight degree by the end sill. Some of the reduced length is produced by the chute blocks in the upstream end of the basin.

For Basin III, the same procedure was followed with chute blocks and baffle piers in the basin. After the baffle piers had been moved upstream as far as possible, without jets of water rising from the piers to penetrate the water surface, the end sill was moved upstream to the position which produced the same degree of performance at the end sill obtained for Basin II. In this case, the baffle piers "force" the jump to occur in a shorter distance. The end sill in Basin III is therefore not a necessary item in the sense that it forces the jump to occur. Only when the tail water is lower than conjugate depth does the sill help to hold the jump in the basin.

Since the sill is subjected to greater flow action in Basin II than in Basin III, the dentated sill dimensions are more critical than the flat sill of Basin III. The height of the sill in Basin III is not critical; however, the sloping face should be sufficiently long to provide directional effect to the flow leaving the basin. A steep sill tends to produce a more violent ground roller which may

ft bed material into the current. Fine material will then be carried away by
ie upper water currents. A flatter sill produces a mild ground roller and
ear neutral conditions in the bed material just downstream from the end sill,
esulting in minimum scour and surface waves in the downstream channel.
'he recommended end sill therefore was chosen to provide optimum per-
ormance, a smooth water surface, and minimum scour.

In Figure A, Mr. Rand shows a range of basin lengths obtained by the uthors and others and relates them to a coefficient K. He summarizes the esults by stating "* * * there is no basin length that is the only correct one." 'he authors believe that there is only one correct length—the shortest one iat can be used safely. This the authors have shown in their length charts. .s stated in the "Introduction" to Paper 1401, many tests have been made to etermine basin dimensions, but when groups of tests which combine the udgment and requirements of several experimenters are lumped together, ie conclusions are often misleading and sometimes contradictory. Since udgment cannot be eliminated from a hydraulic jump testing program, it is ecessary for the reader to decide from the description of the tests whether ie results are dependable. If they are, he should accept the findings entirely. .ny attempt to mix the features of a stilling basin recommended by one group f experimenters with those recommended by another will lead to difficulty.

The average tail-water depth 0.99 D_2, given in Table 2, was not the reason or recommending full tail-water depth, D_2, for Basin III. It was coincidental iat the recommended value and the average value in Table 2 were the same. 'ests on Basin III showed that performance was far better at conjugate depth ian at the minimum tail-water depth, 0.83 D_2, shown in Figure 11. The rea-ons for using full conjugate depths in Basin III are fully explained in Paper 403 under the heading "Tail-water Depths."

Mr. Rand asks the question "Could not more flexibility (in sill height) be rovided?" The authors found in these tests and in tests of other stilling asins not reported that higher end sills, particularly those with vertical aces, tend to increase the wave problem and do not help the scour problem. ills which tend to increase the ground roller action circulate more of the roded material near the sill. Material is continually being packed against ie sill with the excess being carried downstream to be recirculated. The uthors believe that the grinding action will damage the concrete by abrasion. ills which are too low produce greater scour depth close to the end of the pron. The sill height therefore was chosen to produce sufficient lift to re-uce scour but not enough to produce grinding or waves in the downstream hannel.

Mr. Blaisdell shows that the authors' energy loss curves based on their est data fit the theoretical energy loss curves well. Since the basin lengths or Basins II, III, and IV were chosen so that practically all of the jump action ccurs within the basin, the loss curves of Figures 8 and 8A may be applied Basins I through IV. In Mr. Blaisdell's SAF basin, Figure 8, much of the ction occurs downstream from the end sill, leaving a considerable portion of ie hydraulic losses to occur in the downstream channel.

During the testing program to develop Basins I through IV, the authors onstructed and tested several SAF basins. In their judgment and that of other esign engineers in the Bureau of Reclamation, the overall factor of safety in-erent in the SAF design was too small for the relatively large structures de-igned in the Denver office of the USBR. In fact, Mr. Blaisdell states in his aper on the SAF basin "the safety factor incorporated in the design of the

SAF stilling basin is low, but it is felt to be adequate." For the relatively small structures for which Mr. Blaisdell designed the SAF basin this is true.

Experiences in the field with full-sized structures have demonstrated to the authors that tail-water elevation is often the most unpredictable feature in a basin. Experience has shown that the tail water may be several feet lower than anticipated because of riverbed retrogression or for other reasons, or that the time required to establish the tail-water elevation may be longer than anticipated. Considering these factors, it is necessary that a stilling basin contain a built-in factor of safety against sweepout which is greater than the probable error in determining tail-water elevation. In remote areas, it is sometimes necessary to compute the tail-water elevation from incomplete records. In these instances, it is essential that a greater factor of safety than provided in the SAF basin be used. The choice between the two basins, therefore, may be a matter of judgment despite the fact that extensive data are presented to prove the value of both the SAF and Basins I-VI.

The number of baffle piers to be installed in Basin III is not as critical as Mr. Blaisdell indicates in his example. Changing the percent of space occupied by the baffle piers in Basin III by several points would not adversely affect performance and this leeway gives the designer the opportunity to obtain the desired number and arrangement of piers.

The authors would like to thank the discussers of this series of papers for their interest and helpful suggestions. Many of the comments made will be of value in future investigations of this type and in the continuation of this development program.

Corrections

Paper	Page	Correction
1404	7	Near the end of the second paragraph, the word reductions should be changed to reduction.
1405	20	The name D. D. Handlanb should be B. D. Hindland.
1406	6	Bibliography #5 the name Kinsvater should be Kindsvater.

HYDRAULIC DESIGN OF STILLING BASINS: HYDRAULIC JUMP ON A HORIZONTAL APRON AND HYDRAULIC JUMP ON A SLOPING APRON[a]

Closure by J. N. Bradley and A. J. Peterka

J. N. BRADLEY[1] and A. J. PETERKA.[2]—Mr. Vega has used the authors' data to attempt to substantiate equations derived by Mr. F. J. Dominquez and to present an alternate method of expressing the length and depth of hydraulic jumps on sloping aprons. Although the general mathematical theory appears to be in order, it was necessary for Mr. Vega to make certain assumptions to convert the basic equations into usable forms. Apparently, the assumptions were not entirely valid, since the results actually measured by the authors cannot be reproduced from the final equations and charts developed by Mr. Vega. According to Mr. Vega, the percentage of error of his equation (8) from the actual values measured by the authors was 5.07 percent; however, individual values varied as much as 17.7 percent as shown in his Table 1. An inspection of his Figures 4 and 5 shows that the curves only approximate the points, particularly the curve of Figure 5. In combining the two curves, Mr. Vega has, in effect, averaged the jump lengths on all sloping aprons and has eliminated the differences in jump length found by the authors for various apron slopes and shown in Figure 32. Mr. Vega's statement that "* * * the length of the jump is independent of the slope * * * " is therefore incorrect and the curve of his Figure 6 gives only an approximate answer. Similarly, the curves of Figure 7, which give the initial and final depths before and after the jump, are also approximate. Over the midrange of Xo values, the curves are probably sufficiently accurate for ordinary problems but at low values of Xo (which correspond to high values of the Froude number) the curves indicate a deficiency in tail-water depth.

The following example illustrates the typical differences in jump dimensions to be expected from the authors' and Mr. Vega's methods. Assume a spillway with an ogee overflow section to have a discharge coefficient of 3.8 and a unit discharge of 300 second-feet. The head on the crest would be 18.4 feet. Give the jump dimensions for fall heights of 50, 100, and 250 feet for apron slopes tan ϕ of 0, 0.05, 0.10, 0.15, 0.20, and 0.25. The Froude number F for the three heights of fall is 4.15, 6.55, and 11.38, respectively. Using the authors' methods along with Figures 15, 31, and 32, the values in the table below may be computed for the three Froude numbers given above. The values in the table are plotted in Figure 47. The jump length varies considerably,

a. Proc. Papers 1401 and 1405, October, 1957, by J. N. Bradley and A. J. Peterka.

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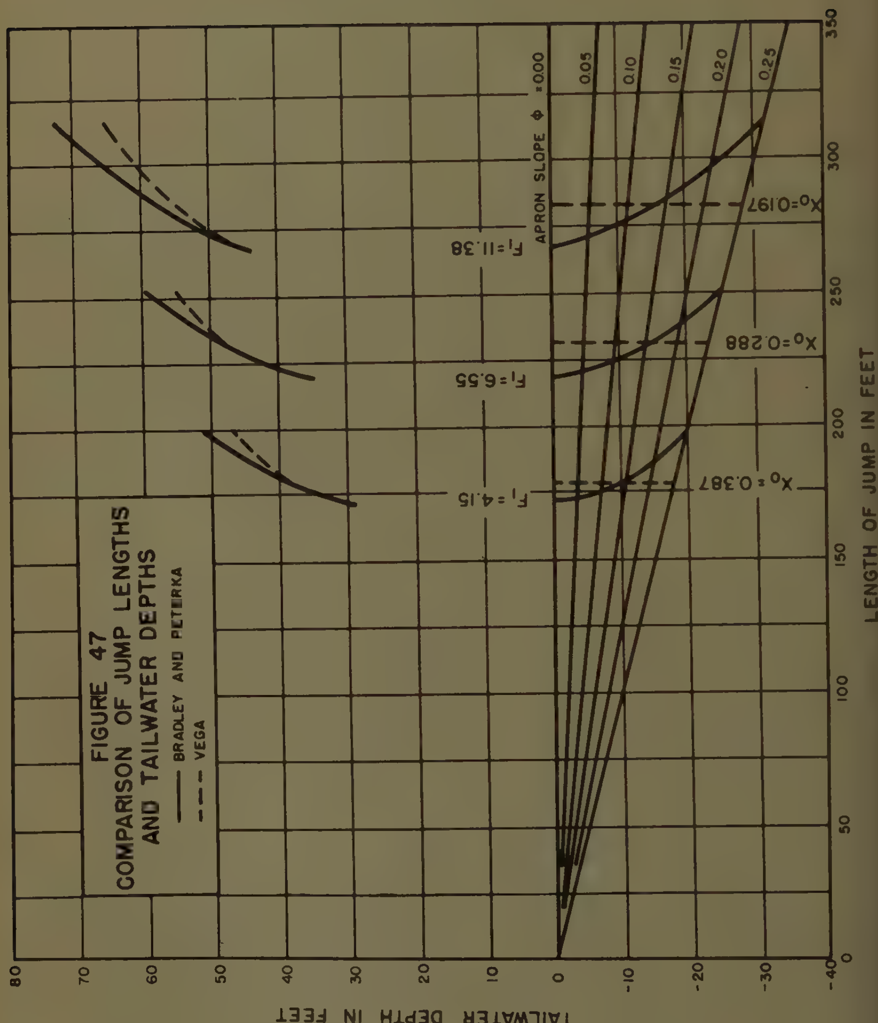

FIGURE 47
COMPARISON OF JUMP LENGTHS AND TAILWATER DEPTHS

becoming longer as the apron slope increases. This is shown by the lower group of curved solid lines which are loci of jump lengths for the range of slopes shown. The necessary tail-water depth also increases as the apron slope increases. The required depths are shown as the upper group of solid line curves, plotted above the apron to which the depth applies.

Using Mr. Vega's methods, the results are somewhat different. For identical flow conditions, Xo is 0.387, 0.288, and 0.197. Figure 6 shows the jump length to be the same for all apron slopes. The jump lengths for the three Xo values have been plotted on Figure 47 (as the three vertical dotted lines) for comparison with the authors' findings. For each F_1 or Xo value, the jump length determined from the equation derived by Mr. Vega is too long for flat slopes and too short for the steep slopes. In the extreme case, the jump length shown by Mr. Vega, is 282 feet compared to 314 feet determined from the authors' methods. In addition, the tailwater depths, indicated in Figure 7 and plotted above the apron end as found by the authors' methods, are deficient by the amount shown in the table and in Figure 47. For the extreme case illustrated, F_1 = 11.4 or Xo = 0.197, for an apron slope of 0.25, the tail-water depth found by Mr. Vega is 7 feet less than the 104.5 feet determined by the authors to be the minimum.

Some of the comments made by Mr. Thomas concerning sloping aprons are questioned by the authors. Mr. Thomas starts his discussion with a statement that the analysis of a sloping apron requires an assumption of jump length, a dimension which is not precisely known. Apparently, Mr. Thomas does not accept the fact that the entire testing program on sloping aprons was essentially devoted to measuring the jump length on various slopes. The results are shown in Figures 32 and 33.

The authors' test apparatus was sufficiently large that some air was present in all the jumps tested. As might be expected, the larger flumes having higher velocities had the most entrained air. No actual air quantities were measured but it is believed that the air quantities were not sufficient to affect the water surface profiles. Further, the authors believe from their experiences with models and prototypes that in the usual prototype structure, the water surface profile will not be changed sufficiently to be of concern. The usual freeboard allowances made on prototype training walls will include the effect of increased air entrainment.

Mr. Thomas states that a gently sloping apron would serve as a "visual warning" to prevent moving the jump off the apron during gate operation and that the gently sloping apron would be superior in this respect to a horizontal apron. The authors are not sure what Mr. Thomas means by "visual warning" but in neither case does the jump suddenly move off the apron.

In both cases, there is a gradual movement of the jump toward the end sill as the sweepout condition is approached. The taller end sill or the baffle piers on the horizontal apron might make the horizontal apron less susceptible to sudden sweepout than a sloping apron. As stated by the authors, the chief advantage of a sloping apron is that in some installations the slope may reduce excavation costs, and not, as Mr. Thomas states, that the sloping apron would serve as a "visual warning" during gate operation.

The authors did not use baffle piers or even chute blocks on the sloping apron because experience has shown that sloping aprons are usually associated with very high dams. The sloping apron without appurtenances of any kind except a small end sill is therefore not limited by entrance velocities which might produce cavitation damage on or adjacent to the chute blocks or

baffle piers. No doubt a sloping apron using appurtenances could be developed. However, there has been no need for this type of structure in the authors' work. The authors regret that Figure A was not included with Mr. Thomas' discussion.

The information presented by Mr. Beck on the Santee Cooper, Petenwell and Castle Rock Dam stilling basins (his Table 1 and Figure 2) is interesting. The results obtained from individual model studies agree quite well with the general curves presented on sloping apron basins. It is not possible to comprehend the work involved in proportioning a sloping apron by trial, even with the aid of an individual model, without actually experiencing it. It is still advisable to verify sloping apron designs for large structures with an individual model, but the generalized curves presented by the authors should eliminate all or the greater part of the trial process.

Notes for Table 12

On ogee crest, discharge coefficient is 3.8, discharge per foot width 300 cfs, head on crest is 18.4 feet.

For 50 ft of fall	V_1 = 55 ft/sec F_1 = 4.15 d_c = 14.08	D_1 = 5.45 ft Xo = 0.387
For 100 ft of fall	V_1 = 74.5 ft/sec F_1 = 6.55 d_c = 14.08	D_1 = 4.02 ft Xo = 0.288
For 250 ft of fall	V_1 = 108 ft/sec F_1 = 11.38 d_c = 14.08	D_1 = 2.78 ft Xo = 0.197

Table 12

Source	Apron slope:	0	0.05	0.10	0.15	0.20	0.25	
Fig 7	X_1 =	2.06	2.53	3.00	3.45	4.06	4.70	Vega for Xo = 0.387
$(X_1)(d_c)$	TW =	29.0	35.6	42.3	48.6	57.2	66.2	
Fig 6	L =	$(L_c = \frac{L_1}{d_c} = 12.6)$ L_1 = 177.5 ft for all slopes						
Fig 32	$\frac{L}{TW}$ =	5.80	4.83	4.12	3.62	3.16	2.78	Bradley and Peterka for F_1 = 4.15
Fig 31	$\frac{TW}{D_1}$ =	5.40	6.50	7.75	9.20	10.80	13.00	
$D_1 \frac{TW}{D_1}$	TW =	29.4	35.4	42.2	50.1	58.9	71.0	
L/TW x TW	L =	170	171	174	181	186	197	
	X_1 =	2.50	3.05	3.58	4.18	4.96	5.70	Vega for Xo = 0.288
	TW =	35.2	43.0	50.4	58.9	70.0	80.2	
	L =	$(L_c = \frac{L_1}{d_c} = 16.3)$ L_1 = 230 ft for all slopes						
	L/TW =	6.13	5.18	4.36	3.82	3.36	2.95	Bradley and Peterka for F_1 = 6.55
	$\frac{TW}{D_1}$ =	8.8	10.50	12.65	15.00	17.55	21.15	
	TW =	35.4	42.2	50.8	60.4	70.5	85.0	
	L =	217	219	222	230	237	251	
	X_1 =	3.10	3.68	4.35	5.08	5.95	6.90	Vega for Xo = 0.197
	TW =	43.7	51.7	61.3	71.6	84.1	97.2	
	L =	$(L_c = \frac{L_1}{d_c} = 20)$ L_1 = 282 ft for all slopes						
	$\frac{L}{TW}$ =	6.05	5.13	4.35	3.80	3.40	3.00	Bradley and Peterka for F_1 = 11.38
	$\frac{TW}{D_1}$ =	15.8	19.0	22.8	26.8	31.3	37.6	
	TW =	44.0	52.8	63.4	74.4	87.0	104.6	
	L =	266	270	275	283	296	314	

THE HYDRAULIC DESIGN OF STILLING BASINS: SMALL BASINS FOR PIPE OR OPEN CHANNEL OUTLETS—NO TAIL WATER REQUIRED (BASIN VI)[a]

Closure by J. N. Bradley and A. J. Peterka

J. N. BRADLEY[1] and A. J. PETERKA.[2]—The impact-type stilling Basin VI has been subjected to a series of tests by Mr. Argue and its performance compared with a more standard type of basin having flared side walls, a depressed bottom, and an end sill. Despite the clearly stated dimensions and capacities given for the impact basin in Table II, Mr. Argue has based most of his comparison tests on an overloaded structure. The maximum capacity of Basin VI, 13 feet wide, is given in Table II as 191 cfs. The test discharge of 330 cfs used by Mr. Argue represents an overload of 73 percent above the recommended capacity. It is not clear to the writers how it would be possible to pass this extra discharge through the basin without water passing over the top of the baffle. Figure 11A shows little if any flow over the baffle. Also, it appears in Figure 11 that the structure was set in a very wide channel with no wing walls connecting the basin to the banks. Induced eddies could then influence the flow pattern leaving the basin by concentrating the outflowing jet into a narrow stream. Mr. Argue has mentioned this as one of the characteristics of Basin VI which he did not like. When side eddies are prevented, as in the recommended installation, Figure 46, this type of flow does not occur.

Mr. Argue states that the impact basin has a "rigid relationship between maximum discharge and basin dimensions and no variation in these dimensions is given with different erodible materials downstream from the structure." Apparently, Mr. Argue did not realize the significance of the upper and lower limit curves of Figure 42 which are illustrated by the photographs of Figure 43. The structure width may be varied between the upper and lower limit values to produce varying outflow velocities. For example, for a discharge of 191 cfs, the width may be varied from 11 to 13 feet, making the discharge per foot of width of basin vary from 17.3 to 14.7 cfs. With the alternate end sill, the unit discharge may be reduced further. The relative outlet velocities may be computed (assuming uniform velocity distribution) to determine whether the higher or lower velocities are more compatible with the bed material encountered at a particular site.

The writers believe that Mr. Argue has placed undue emphasis on full pipe performance. Under ordinary conditions on a structure of this type, the pipe will run full only at the maximum discharge. Even then, it is difficult to fill the pipe unless a carefully designed and constructed entrance is provided.

a. Proc. Paper 1406, October, 1957, by J. N. Bradley and A. J. Peterka.

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(It is possible in a 4-inch-diameter model to have full pipe flow which could not occur in a 54-inch prototype. Field experience indicates that the head on the entrance must exceed four times the pipe diameter and the pipe slope must be less than 1 percent to have a full pipe). The writers' Basin VI is therefore designed to give best performance with open channel flow.

The writers have several comments regarding the basin comparison tests. Mr. Argue states that the most obvious differences between the two structures was the variation in discharge; the impact basin gave 10 percent less maximum capacity. It should be understood that the purpose of the impact basin is not to produce a maximum discharge but rather to dissipate and distribute energy effectively for a particular discharge or range of discharges. The fact that the impact basin reduced the maximum discharge—an overload of 73 percent—by 10 percent could hardly be called a defect.

The statement by Mr. Argue, "The hanging baffle had no effect until the flow reached 220 cfs," is probably an inadvertently worded sentence. It is difficult to visualize a discharge of 220 cfs passing through a 54-inch-diameter pipe at a velocity of about 20 feet per second producing no effect when it strikes a vertical wall. In the writers' tests, the flow striking the baffle produced violent eddy action upstream from the baffle. Flow was forced out from beneath the baffle and the end sill produced sufficient resistance to the flow to produce fairly uniform distribution of flow into the downstream channel. Figure 43 illustrates the action in the model and Figure 46 shows performance typical of several prototype structures which have been observed in the field.

It is possible that the overloading of Basin VI prevented proper action of the water in the limited space upstream from the vertical baffle, making it appear that the baffle had little effect on the flow. The increased head above the baffle would also increase the velocity beneath the baffle. These effects could account for the flow not filling the 45 degree expansion at the end sill in Mr. Argue's tests. In the writers' tests, when the quantity of water recommended for the structure was discharged at the recommended velocity, the 45 degree alternate sill was a distinct improvement to the basin.

In regard to the comparison of scour patterns, the statement of a few basic facts will probably clarify the detailed comparisons Mr. Argue has made. There is no magic quality of an energy dissipator which can make it clearly superior to all other energy dissipators for all operating conditions. Hydraulic structures which will handle an overload usually do not work as well for small discharges and vice versa. A design which has been developed for use with riprap protection will probably produce scour if the riprap is not used. In short, unless a structure is tested and used under the conditions for which it was developed, it may not show to best advantage. The writers believe that Basin VI constructed and used according to the recommendations will outperform a basin of the same general size which does not contain a baffle or impact surface. In fact, Figures 2A, 2B, 3A, 3B, 4A, 4B, 5A and 5B show the impact-type basin to be superior or equal in performance with the P. W. D. structure for discharges up to or slightly above the design discharge given for Basin VI.

Mr. Argue's remarks concerning the need for a cutoff wall at the end of the basin are correct. Because of undersized riprap, one prototype structure subjected to near maximum flows was slightly undermined. As a protection against scour of this type in the future, the end wall of the basin has been extended downward (several feet, depending on the type of material) to provide a cutoff wall. However, on structures where adequate riprap protection can be provided, there is no need for this additional protection.

PROCEEDINGS PAPERS

ne technical papers published in the past year are identified by number below. Technical-division sorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air sport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hy- ics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil anics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors , divisions. Papers sponsored by the Board of Direction are identified by the symbols (BD). For and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 ary 1956) papers were published in Journals of the various Technical Divisions. To locate papers in ournals, the symbols after the paper numbers are followed by a numeral designating the issue of a par- r Journal in which the paper appeared. For example, Paper 1449 is identified as 1449 (HY 6) which ates that the paper is contained in the sixth issue of the Journal of the Hydraulics Division during 1957.

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EMBER: 1426(SM4), 1427(SM4), 1428(SM4), 1429(SM4), 1430(SM4)[c], 1431(ST6), 1432(ST6), 1433(ST6), 34(ST6), 1435(ST6), 1436(ST6), 1437(ST6), 1438(SM4), 1439(SM4), 1440(ST6), 1441(ST6), 1442(ST6)[c], 43(SU2), 1444(SU2), 1445(SU2), 1446(SU2), 1447(SU2), 1448(SU2)[c].

EMBER: 1449(HY6), 1450(HY6), 1451(HY6), 1452(HY6), 1453(HY6), 1454(HY6), 1455(HY6), 1456(HY6)[c], 57(PO6), 1458(PO6), 1459(PO6), 1460(PO6)[c], 1461(SA6), 1462(SA6), 1463(SA6), 1464(SA6), 1465(SA6), 66(SA6)[c], 1467(AT2), 1468(AT2), 1469(AT2), 1470(AT2), 1471(AT2), 1472(AT2), 1473(AT2), 1474(AT2), 75(AT2), 1476(AT2), 1477(AT2), 1478(AT2), 1479(AT2), 1480(AT2), 1481(AT2), 1482(AT2), 1483(AT2), 84(AT2), 1485(AT2)[c], 1486(BD2), 1487(BD2), 1488(PO6), 1489(PO6), 1490(BD2), 1491(BD2), 1492(HY6), 93(BD2).

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CH: 1560(ST2), 1561(ST2), 1562(ST2), 1563(ST2), 1564(ST2), 1565(ST2), 1566(ST2), 1567(ST2), 1568 W2), 1569(WW2), 1570(WW2), 1571(WW2), 1572(WW2), 1573(WW2), 1574(PL1), 1575(PL1), 1576(ST2)[c], 77(PL1), 1578(PL1)[c], 1579(WW2)[c].

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: 1658(AT1), 1659(AT1), 1660(HY3), 1661(HY3), 1662(HY3), 1663(HY3), 1664(HY3), 1665(SA3), 1666 L2), 1667(PL2), 1668(PL2), 1669(AT1), 1670(PO3), 1671(PO3), 1672(PO3), 1673(PL2), 1674(PL2), 1675 O3), 1676(PO3), 1677(SA3), 1678(SA3), 1679(SA3), 1680(SA3), 1681(SA3), 1682(SA3), 1683(PO3), 1684 Y3), 1685(SA3), 1686(SA3), 1687(PO3), 1688(SA3)[c], 1689(PO3)[c], 1690(HY3)[c], 1691(PL2)[c].

: 1692(EM3), 1693(EM3), 1694(ST4), 1695(ST4), 1696(ST4), 1697(SU2), 1698(SU2), 1699(SU2), 1700(SU2), 01(SA4), 1702(SA4), 1703(SA4), 1704(SA4), 1705(SA4), 1706(EM3), 1707(ST4), 1708(ST4), 1709(ST4), 1710 T4), 1711(ST4), 1712(ST4), 1713(SU2), 1714(SA4), 1715(SA4), 1716(SU2), 1717(SA4), 1718(EM3), 1719 M3), 1720(SU2), 1721(ST4)[c], 1722(ST4), 1723(ST4), 1724(EM3)[c].

UST: 1725(HY4), 1726(HY4), 1727(SM3), 1728(SM3), 1729(SM3), 1730(SM3), 1731(SM3), 1732(SM3), 1733 O4), 1734(PO4), 1735(PO4), 1736(PO4), 1737(PO4), 1738(PO4), 1739(PO4), 1740(PO4), 1741(PO4), 1742 O4), 1743(PO4), 1744(PO4), 1745(PO4), 1746(PO4), 1747(PO4), 1748(PO4), 1749(PO4).

EMBER: 1750(IR3), 1751(IR3), 1752(IR3), 1753(IR3), 1754(IR3), 1755(ST5), 1756(ST5), 1757(ST5), 58(ST5), 1759(ST5), 1760(ST5), 1761(ST5), 1762(ST5), 1763(ST5), 1764(ST5), 1765(WW4), 1766(WW4), 67(WW4), 1768(WW4), 1769(WW4), 1770(WW4), 1771(WW4), 1772(WW4), 1773(WW4), 1774(IR3), 1775 R3), 1776(SA5), 1777(SA5), 1778(SA5), 1779(SA5), 1780(SA5), 1781(WW4), 1782(SA5), 1783(SA5), 1784 R3)[c], 1785(WW4)[c], 1786(SA5)[c], 1787(ST5)[c], 1788(IR3), 1789(WW4).

OBER: 1790(EM4), 1791(EM4), 1792(EM4), 1793(EM4), 1794(EM4), 1795(HW3), 1796(HW3), 1797(HW3), 98(HW3), 1799(HW3), 1800(HW3), 1801(HW3), 1802(HW3), 1803(HW3), 1804(HW3), 1805(HW3), 1806 Y5), 1807(HY5), 1808(HY5), 1809(HY5), 1810(HY5), 1811(HY5), 1812(SM4), 1813(SM4), 1814(ST6), 1815 T6), 1816(ST6), 1817(ST6), 1818(ST6), 1819(ST6), 1820(ST6), 1821(ST6), 1822(EM4), 1823(PO5), 1824 M4), 1825(SM4), 1826(SM4), 1827(ST6)[c], 1828(SM4)[c], 1829(HW3)[c], 1830(PO5)[c], 1831(EM4)[c], 1832(HY5)[c]

iscussion of several papers, grouped by divisions.